Math 218 **Final Exam** **Spring 1998**

INSTRUCTIONS. Every numerical answer should be simplified to a fraction or decimal. You must show your work and justify your methods to obtain full credit. Use the continuity correction wherever it is appropriate. If you can't do one part of a problem but need that answer later, guess an answer and use that guess for the later part. The exam is worth a total of 200 points.

Problem 1. (20 points) A company with 1,000 employees has the following distribution of salaries for three age groups:

	under 30 years	between 30 and 50	over 50 years
under $50,000	200	250	100
over $50,000	50	150	250

(a) Find the probability that a randomly chosen employee is at least 30 years old.
(b) Find the probability that a randomly chosen employee is under 30 given the employee's salary is over $50,000.
(c) Find the probability that a randomly chosen employee earns over $50,000 given the employee is under 30.
(d) Determine whether the events that a randomly chosen employee is "aged under 30" and "earns over $50,000" are independent. Remember to justify your answer.

Problem 2. (20 points) In a certain country 3 out of every 50 inhabitants are infected with malaria. A test is used to detect malaria; however the test is not always accurate. For people without the disease the test gives a false result 1% of the time. For people with the disease the test gives a false result 2% of the time.

(a) Draw the probability tree.
(b) Find the probability that a randomly selected inhabitant of the country is not infected.
(c) Find the probability that a randomly selected inhabitant of the country tests positive (i.e. the test indicates that the person has the disease.)
(d) Find the probability that a randomly selected inhabitant of the country who tested positive is infected.

Problem 3. (20 points) A furniture retailer receives a shipment of sixteen sofas from a factory. Because inspection of each individual sofa is expensive and time-consuming, the retailer has the policy of inspecting a random sample of six items from the shipment. The retailer will accept delivery of the shipment if at most one sampled sofa in the shipment is defective.

(a) Suppose that the factory has a 22% defective rate when manufacturing sofas. What is the probability that a shipment of sixteen sofas will be accepted?
(b) Suppose instead that the manufacturer knowingly sends a shipment with four defective sofas and twelve good ones. What now is the probability that the shipment will be accepted?

Problem 4. (20 points) A company produces packages of paper clips. The number of clips per package varies, with probability distribution as follows.

Number of clips	48	49	50	51	52
Probability	.05	.25	.35	.20	.15

(a) Find the expected value and standard deviation for the number of clips in a package.
(b) The cost (in cents) of producing a package of paper clips is $27 + 0.7X$ where X is the number of clips in the package. The revenue from selling the package is $1.19. Find the expected value and standard deviation of the profit (i.e. revenue minus cost) per package.
(c) The paper clips are shipped in cartons of 240 packages. Find an approximate value for the probability that a carton contains more than 12,050 paper clips.

Problem 5. (10 points) Suppose that 55% of transportation companies have a debt to equity (D/E) ratio over 150%. A random sample of 120 transportation companies is taken. Find an approximate value for the probability that at least 70 and at most 80 of them have a D/E ratio over 150%.

Problem 6. (10 points) A potential investor has three investment opportunities with uncertain returns. The return on Investment A is normally distributed with a mean of $60 and a standard deviation of $10. The return on Investment B is normally distributed with a mean of $75 and a standard deviation of $20. The return on Investment C is uniformly distributed between $30 and $110.

(a) Which investment has the highest expected return?
(b) What is the probability that the return on investment B is under $60?
(c) What is the probability that the return on investment C is under $60?

Problem 7. (15 points) Assume the amount of coconut milk contained in a fresh coconut has a normal distribution with a mean of 12 ounces and a standard deviation of 7 ounces. A Hawaiian restaurant needs 160 ounces of fresh coconut milk each day.

(a) Suppose that restaurant purchases 14 coconuts each day. What is the probability that the 14 coconuts together contain at least 160 ounces of milk?

(b) Find the smallest number of coconuts which the restaurant should purchase each day so that the probability it runs out of milk is less than 5%.

Problem 8. (15 points) A random sample of size 25 is chosen from a normal population. The sample mean is 96.2 and assume that the population standard deviation is known to be 15.0.

(a) Calculate a 95% confidence interval for the population mean.
(b) Recalculate the 95% confidence interval for the population mean assuming instead that the population standard deviation is unknown and that the sample standard deviation is 19.7.

Problem 9. (20 points) A ratings company wishes to estimate the proportion p of all TV viewers that watch a particular program. A preliminary random sample of 400 viewers is taken and it is found that 60 of the viewers in the sample are watching the program.

(a) Determine a 99% confidence interval for the proportion p.
(b) Based on the preliminary sample, determine how large a sample is needed to estimate p to within ± 0.03 with 99% confidence.

Problem 10. (25 points) Subliminal advertising is based on the idea that consumer behavior can be influenced by subtle images aimed at a person's subconscious. To test whether subliminal advertising has any effect at all, "quick-cuts" of a variety of images – hot dry deserts, cool refreshing waterfalls, etc. – were spliced into the recent academy award winning film Titanic. The researchers were looking for any change what-so-ever in the amount of soft drink purchased during the showing of the film. In the theater where the testing was being done, an average of 50 gallons of soft drink had been sold during showings of the usual version of Titanic. (Assume the amount of soft drink has a normal distribution. Assume also that the audience size remains constant.)

(a) Formulate appropriate null and alternative hypotheses to test whether this subliminal advertising has any effect.
(b) Data will be taken at 9 screenings of the newly edited version of the Titanic. Choose a suitable test statistic and rejection rule to test your null hypothesis against your alternative hypothesis at the 5% level of significance.
(c) The number of gallons of soft drink purchased during each of the 9 screenings of the newly edited version of the Titanic were

62 56 48 53 47 55 54 53 58

Determine whether the null hypothesis should be rejected at the 5% level of significance. Explain your reasoning.

Problem 11. (25 points) Suppose that the best drug treatment currently available for a particular disease is effective on 60% of the patients who use it. A pharmaceutical company comes up with a new drug treatment. In order to test whether the new treatment will be effective on a larger proportion of patients, it conducts clinical trials on a randomly chosen sample of 40 patients with the disease.

(a) Formulate appropriate null and alternative hypotheses to test whether the new treatment is effective in a higher proportion of cases.
(b) Choose a suitable test statistic and rejection rule to test your null hypothesis against your alternative hypothesis at the 10% level of significance.
(c) Suppose that the new drug treatment is effective on 29 of the 40 patients. Determine whether the null hypothesis should be rejected at the 10% level of significance. Explain your reasoning.
(d) Find the P-value for the data in this test. Your answer should be either an exact value, or else the smallest interval which can be determined from the tables provided.
(e) Suppose instead that a different clinical trial of the same drug had resulted in a P-value of 0.0262. Determine at which of the following levels of significance the null hypothesis should be rejected. Circle all that apply.

0.5% 1% 2.5% 5% 10%

Answers to Spring 1998 Math 218 Final Exam

1.: (a) 3/4; (b) 1/9; (c) 1/5; (d) No.
2.: (b) 0.94;(c) 0.0682; (d) 0.8622.
3.: (a) 0.6063; (b) 0.5110.
4.: (a) 50.15, 1.1079; (b) 56.9 cents, 0.77 cents; (c) using the continuity correction
$P(T > 12050) = P(T \geq 12050.5) \approx P(Z \geq 0.84) = 0.2005$; otherwise $P(T > 12050) \approx P(Z > 0.82) = 0.2061$ or $P(T \geq 12051) \approx P(Z \geq 0.87) = 0.1922$.
5.: 0.2572.
6.: (a) B; (b) 0.2266; (c) 3/8.
7.: (a) 0.6217; (b) 18.
8.: (a) from 90.32 to 102.08; (b) from 88.07 to 104.33.
9.: (a) from 0.104 to 0.196; (b) 941.
10.: (a) $H_0 : \mu = 50$, $H_a : \mu \neq 50$; (b) $t = (\overline{X} - 50)/(s/3)$, reject H_0 if $|t| > 2.306$; (c) $t = (54 - 50)/(4.637/3) = 2.588$, so reject H_0.
11.: (a) $H_0 : p \leq 0.6$, $H_a : p > 0.6$; (b) $Z = (\hat{p} - 0.6)/\sqrt{(0.6 \times 0.4/40)}$, reject if $Z > 1.282$; (c) $Z = 1.614$, so reject H_0; (d) 0.0537; (e) 5% and 10%.