

MATH 218 FINAL EXAMINATION: FALL 2010

Name (printed) _____ Student ID _____

Signature (handwritten) _____

DIRECTIONS. Fill in your name, signature, and student ID number on the lines above; then checkmark your lecture time and discussion time in the space provided below. **Do not OPEN the test booklet until instructed to do so.** After you are so instructed, make sure all 12 pages are present (not including tables, which are distributed separately). *Note that the separately distributed sheet of tables is printed on both sides.*

On this examination, you may use a calculator and *one* $8\frac{1}{2}$ by 11-inch sheet of *handwritten* notes (both sides may be written on). No books or other notes are permitted.

Show your work — Numerical answers alone are not sufficient; you **MUST** indicate how you derived them.

When an answer box is provided, copy your answer into that box. Numerical answers should be evaluated as either decimals or fractions. When submitting a numerical answer as a decimal, use the number of decimal places warranted by the data. Be sure to include the appropriate units (for example, feet, \$, pounds, mph, etc.).

Check one lecture time and one discussion time:

9:00 MWF (Emerson) 11:00 MWF (Emerson) 10:00 MWF (Lytvak)

TuTh Discussion 8:00 9:00 10:00 11:00 12:00 1:00

_____ *Do not write on this page below this line!* _____

1 (24 pts)		7 (14 pts)	
2 (18 pts)		8 (18 pts)	
3 (18 pts)		9 (15 pts)	
4 (14 pts)		10 (18 pts)	
5 (20 pts)		11 (23 pts)	
6 (18 pts)			
(112 pts)		(88 pts)	

Total Points

Problem 1. (24 points) Color blindness is a sex-linked, inherited condition that is much more common among men than women. Suppose that 5% of all men and 0.4 % of all women are color-blind. Also assume that half of the population are men and half are women.

- (a) Draw a probability tree describing this situation. Be sure to include the labels of events, probabilities, conditional and joint probabilities.

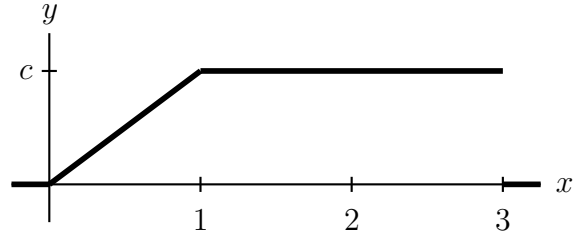
- (b) A person is chosen at random and found to be color-blind. What is the probability that the person is male?

- (c) Are events “person is color-blind” and “person is male” independent? Explain.

- (d) 1000 people are randomly selected. Find the probability that at most 20 of them are color-blind. State whether your answer is exact or approximate. If you use an approximation, justify why it is valid to do so.

Problem 2. (18 points) Consider a random variable X with density function $f_X(x)$ which depends on some constant c . The formula and graph of $f_X(x)$ are given below:

$$f_X(x) = \begin{cases} 0 & x \leq 0; \\ cx & 0 < x \leq 1; \\ c & 1 < x \leq 3; \\ 0 & 3 < x. \end{cases}$$



(a) Find c .

(b) Find $P(X \leq 1.3)$.

(c) Find $E(X)$.

Problem 3. (18 points) Buy More™ electronic stores sells DVD players and home entertainment systems. Let X be the number of DVD players and Y the number of home entertainment systems sold by one salesperson in a week. The table for the joint distribution of X and Y is given below:

		Y			
		0	5	10	
X	0	.05	0	0	
	5	.15	.4	.09	
	10	0	.25	.06	

- (a) Find the probability that during the next week a salesperson will sell more DVD players than home entertainment systems.

(a)

- (b) Find $\text{Cov}(X, Y)$.

(b)

- (c) Find $E(X + 2Y - 3)$.

(c)

- (d) Find $\text{Var}(X + 2Y - 3)$.

(d)

Problem 4. (14 points) It is known that only 20% of biotech firms survive for more than five years. Suppose that 10 biotech firms are randomly chosen and each one survives for more than five years independently of the others.

- (a) What is the expected number of firms that will survive for more than five years?

- (b) What is the standard deviation of the number of firms that will survive for more than five years?

- (c) Find the probability that exactly 2 firms survive for more than five years.

- (d) Find the probability that no more than 2 firms survive for more than five years.

Problem 5. (20 points) A Las Vegas roulette wheel has 38 numbered slots. Roulette wheels are calibrated such that each outcome is equally likely. To play roulette, you choose a number; you win \$35 if the ball ends up in that slot and lose \$1 otherwise. Let X denote your winnings/losses when you play once.

(a) Write down the probability distribution of X . *Box your answer.*

(b) You play five times. Find the probability that you will not lose money.

(b)

(c) You decide to play once a minute for 12 hours a day for the next week, a total of 5040 times. Find the probability that you will not lose money with this amount of roulette playing.

(c)

Problem 6. (18 points) A monkey is pounding away randomly on a word processor. On average, the monkey produces 2 real words per minute, and the rest are nonsensical. Assume that the number of real words generated by the monkey follows a Poisson distribution.

- (a) What is the probability that the monkey produces exactly 5 real words in the next 3 minutes?

- (b) How long should the monkey expect to wait until producing the next real word?

- (c) What is the probability that the time between 2 consecutive real words produced by the monkey is less than 1 minute?

Problem 7. (14 points) In some states the law requires drivers to turn on their headlights when driving in the rain. A highway patrol officer believes that only between one-quarter and one-third (inclusive) of all drivers follow this rule. He wants to construct a 98% confidence interval for the proportion of all drivers who turns on their headlights when driving in the rain.

- (a) How many cars driving in the rain does he need to examine so that this 98% confidence interval for the proportion has a width of no more than 0.07?

(a)

- (b) He randomly samples 300 cars driving in the rain and finds that 90 of them have their headlights turned on. Based on this information, find the 98% confidence interval for the proportion of all drivers who turns on their headlights when driving in the rain.

(b)

Problem 8. (18 points) USC students were asked which college sport they are the biggest fan of. They had to choose exactly one sport. In one class of 30 students, there were 25 football fans and 5 basketball fans. Suppose we choose a random sample of 6 students from this class.

(a) How many football fans do we expect there to be in the sample?

(b) Find the probability that there are exactly 5 football fans in the sample.

(c) Suppose that the first 3 students chosen in the sample are football fans. Find the probability that there are exactly 5 football fans in the sample.

Problem 9. (15 points) In order to determine the height of Hobbits, a random sample of 4 Hobbits was chosen. Their heights (in inches) were

43, 42, 40, 38.

Assume that heights of Hobbits are normally distributed.

- (a) Find the 95% confidence interval for the height of Hobbits.

(a)

- (b) Many sources give the average height of Hobbits as 42 inches. Is 42 inches in the above confidence interval?

Yes

No

- (c) Which of the following is true? Circle the best answer. *You do not need to show your work, but if you do, you may receive partial credit even if your answer is incorrect.*

- i. We can be 95% certain that the average height of Hobbits is 42 inches.
- ii. We can be 95% certain that the average height of Hobbits is *not* 42 inches.
- iii. The above data is consistent with the belief that the average height of Hobbits is 42 inches.
- iv. About 95% of Hobbits are shorter than 43 inches.

Problem 10. (18 points) By design, scores on the SAT Math subject test are normally distributed with a mean of 500 and a standard deviation of 100. An SAT preparation class claims it will increase scores.

Let μ be the mean score of students who took the preparation class. The null and alternative hypotheses are given below:

$$H_0: \mu = 500$$

$$H_a: \mu > 500$$

A random sample of 60 students who took the class was chosen. Their average score was 530.

(a) Determine the test statistic and evaluate it numerically.

(a)

(b) Find the p -value of your statistic.

p -value =

(c) Circle all levels of significance that we can reject the null hypothesis at.

10% 5% 2.5% 1% 0.5%

(d) Based on the above analysis, using the 5% significance level, would you recommend this SAT preparation class? Briefly explain in clear, non-technical language.

Problem 11. (23 points) Historically, the average assembly time for a piece of office furniture was 1 hour. Recently, office furniture producers introduced new instructions for their furniture. In order to determine if the average assembly time has changed in any way, 5 office furniture stores are surveyed and they report that the assembly times of this piece of furniture (in minutes) are

55, 65, 85, 55, 60.

Assume that the assembly time is normally distributed.

- (a) Formulate appropriate null and alternative hypotheses.

H_0 :

H_a :

- (b) Determine the test statistic and evaluate it numerically.

(b)

- (c) Formulate the rejection rule at the 1% significance level. Based on that decide whether the company claim can be rejected.

(c)

- (d) Estimate the p -value of this test (circle one). Show your work.

- i. less than 0.01
- ii. between 0.01 and 0.05
- iii. between 0.05 and 0.1
- iv. greater than 0.1