MATH 218 FINAL EXAM December 16, 2009

Last Name: ______________________ **First Name:** ________________________

Student ID Number: ________________ **Signature:** ________________________

Circle your instructor's name and session time:

Dumett (2pm) **Fulman** (1pm) **Kim** (10am) **Lin** (11am), (12noon)

Lytvak (11am), (12noon), (2pm) **Mancera** (1pm), (2pm) **Piterbarg** (9am)

Polunchenko (10am) **Rosen** (1pm) **Yin** (10am),(11am)

INSTRUCTIONS

- On this examination, you may use a calculator and one $8\frac{1}{2}$ inch by 11 inch sheet of handwritten notes (both sides may be written on). No books or other notes are permitted.
- Answer all eleven problems. Numerical answers alone are not sufficient. **You must indicate how you derived them (show work) to obtain full credit**. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page.
- When an answer box is provided, put your final answer in the box.
- When submitting a numerical answer that is a decimal, use **four decimal places of accuracy after the decimal point or, if warranted by the data, the appropriate smaller number of decimal places.**
- When submitting a numerical answer that is a fraction, reduce it to lowest terms.
- Be sure to include the units in your answer.
- If you can not do part a) of some problem, but you need the answer for part b), then you can get partial credit for showing you know what to do. You could write, "let *p* be the answer to a)," and solve b) in terms of *p*.
- The value of a problem is indicated in parentheses following the problem number. The exam is worth a total of 220 points.

Problem	Value	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
11	20	
Total	220	

Problem 1. (20 points)
In a large community 60% of all people have Visa credit cards and 50% of all people have MasterCard credit cards. 27% of people in this community have MasterCard credit cards but no Visa credit cards. Assume that no one can have more than one card of the same type.

(a) (5 points) Find the probability that a randomly selected person in this community has both Visa and MasterCard credit cards.

(a)

(b) (5 points) Find the probability that a randomly selected person in this community has exactly one credit card (Visa or MasterCard).

(b)

(c) (5 points) Are the events "person has a Visa card" and "person has a MasterCard card" independent? Explain.

(c)

(d) (5 points) Is a randomly selected person more likely or less likely to have a MasterCard card if he/she has a Visa card? Explain.

(d)

Problem 2. (20 points)
There are three bags containing marbles. Two bags contain 5 red and 1 black marbles each and one bag contains 3 red and 3 black marbles. A bag is randomly selected and then two marbles are randomly selected without replacement from that bag.

(a) (10 points) Find the probability that both marbles selected have the same color. (Hint: A probability tree may be useful in this situation).

(a)

(b) (10 points) If both marbles selected are red, what is the probability that they were chosen from the bag originally containing 3 red and 3 black marbles?

(b)

Problem 3. (20 points)

Let X be a random variable with probability distribution defined by the following table:

x	-1	0	1	2
$P_X(x)$	$1/5$	$1/5$	$2/5$	$1/5$

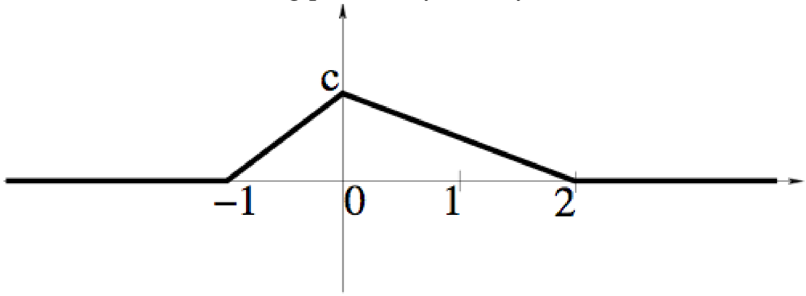
(a) (7 points) Find the conditional probability of X given that X is positive. (i.e. $P_X(x \mid X > 0)$)

(b) (7 points) Let Y be the random variable defined by $Y = 2X + 3$. Find the probability distribution $P_Y(y)$ of Y.

(c) (6 points) Calculate the standard deviation of Y.

Standard deviation =

Problem 4. (20 points)
A continuous random variable X has the following probability density function:



Indeed, the probability density function $f(x)$ is given by

$$f(x) = \begin{cases} 0, & \text{if} \quad x < -1, \\ c(x+1), & \text{if} \quad -1 \le x \le 0, \\ c\left(1 - \frac{1}{2}x\right), & \text{if} \quad 0 \le x < 2, \\ 0, & \text{if} \quad x \ge 2. \end{cases}$$

(a) (6 points) Find the value of the number c.

$c =$

(b) (7 points) Find the probability $P(X \ge 1)$.

$P(X \ge 1) =$

(c) (7 points) Find the cumulative distribution function $F_X(x) = P(X \le x)$ for $-1 \le x \le 0$.

$F_X(x) =$

Problem 5. (20 points)
Within a particular plan, a financial investment firm offers investors a selection of 12 different stocks and 5 different bonds from which to construct their portfolios.

(a) (6 points) If an investor selects a portfolio of 10 distinct items at random, find the probability that the portfolio contains exactly 6 stocks and 4 bonds.

(a)

(b) (7 points) Each year, the investor selects the 10 distinct items for the portfolio at random. Find the probability that in exactly 2 of the next 3 years, the investor's portfolio contains exactly 6 stocks and 4 bonds. If you did not get an answer to part (a), assume that the answer of part (a) is 0.2.

(b)

(c) (7 points) For the next 30 years, find the probability that the investor's portfolio contains exactly 6 stocks and 4 bonds in at least 10 years but in no more than 20 years. State whether the answer is exact or approximate and explain.

(c)

Problem 6. (20 points)
At a particular bank branch, the number of customers reaching a teller's window is Poisson distributed with a rate of 4 customers per 10-minute period.

(a) (5 points) Find the probability that the time between consecutive customer arrivals at the teller's window is less than 3 minutes.

(a)

(b) (5 points) For the next 5 consecutive customer arrivals, find the probability that at least 2 of the consecutive customer arrivals are less than 3 minutes apart. If you did not get an answer to part (a), assume that the answer of part (a) is 0.8.

(b)

(c) (5 points) For the next 5 consecutive customer arrivals, find the expected number of consecutive customer arrivals occurring less than 3 minutes apart.

Expected value =

(d) (5 points) For the next 5 consecutive customer arrivals, find the standard deviation of the number of consecutive customer arrivals occurring less than 3 minutes apart.

Standard deviation =

Problem 7. (20 points)
For a particular daily flight, airline records indicate that the probability distribution of the number of passengers holding a reservation but not showing up can be approximated by a normal distribution with a mean of 20 passengers and a standard deviation of 5 passengers. To protect against no-shows, the airline overbooks its flights. This means that for this flight using planes holding 100 passengers, the airline will take reservations for up to 112 passengers.

(a) (7 points) On a particular day the airline took reservations for 112 passengers for the flight. What is the probability that every passenger arriving on time for the flight will have a seat?

(a)

(b) (7 points) For the next 7 daily flights, what is the probability that the total number of no-shows is greater than or equal to 150 passengers?

(b)

(c) (6 points) What should be the new number of reservations taken on each flight, in order that a passenger holding a reservation and showing up on time will have a 97.72% probability of getting a seat on that flight?

(c)

Problem 8. (20 points)

(a) (10 points) Find a 95% confidence interval of the population mean FICO score (credit rating) of working adults in the United States based on a random sample of 50 applications for credit cards with a sample mean of 677. Assume the population standard deviation σ is known to be 68.

Confidence interval:

(b) (10 points) We want a 90% confidence interval such that the sample mean is within ± 3 of the population mean. Find the minimum required sample size.

Sample size:

Problem 9. (20 points)
A random sample of 65 individuals is selected in order to determine what proportion of the population has diabetes. Of the 65 individuals, 13 are found to have the disease.

(a) (10 points) Find a 90% confidence interval for the proportion of people with diabetes in the general population.

Confidence interval:

(b) (10 points) Suppose you want to be sure that the proportion is estimated to within ± 0.08 with 95% confidence. How many more individuals should be selected at random from the population in order to guarantee this?
(Use the point estimate obtained in part (a) to obtain a sample size)

Additional sample size:

Problem 10. (20 points)

Based only on anecdotal evidence, vaccination clinics have been telling people receiving the swine flu vaccination that on average it takes 10 days from when you received the shot until you are fully immune to the virus. However, researchers are interested in seeing if the 10 day window is in fact supported by statistical evidence.

(a) (5 points) Provide null and alternative hypotheses that researchers can use to test whether immunity develops either more rapidly or more slowly than is commonly thought.

H_0:	H_a:

(b) (5 points) There are no existing guidelines for immunological studies that would provide the researchers with any reliable estimate for the population variance in how long it takes to develop antibodies for this strain of flu. Consequently the variance will have to be estimated from clinical data. Researchers took daily blood samples from a randomly selected group of 9 people receiving the swine flu vaccine to determine when an adequate level of antibodies had developed. They obtained the following data (in days): 10, 8.5, 6, 7.5, 9, 9.5, 11.5, 9.5, and 5. If we assume that immunity waiting periods are normally distributed, and use a 5% level of significance, provide a test statistic and a rejection rule, that the researchers can use in testing the hypotheses you provided in part (a).

Test statistic (formula):

Its value:

Rejection rule:

(c) (5 points) Based on the data in part (b), can the null hypothesis be rejected at the 5% level of significance? You must justify your answer to receive any credit.

Decision (circle one): Do not reject H_0 Reject H_0

(d) (5 points) What can you say about the p-value for this test (circle one)? Show your work.

(1) The p-value is between 0.025 and 0.05
(2) The p-value is between 0.05 and 0.1
(3) The p-value is between 0.1 and 0.25
(4) None of the above.

Problem 11. (20 points)
A small company uses a parcel service to ship packages of special cheeses ordered as gifts. In the past 90% of the orders were delivered on time. More recently, due to new efforts the company claims that over 90% of the orders are delivered on time. A sample of 1000 deliveries are studied and there are 915 orders delivered on time.

(a) (3 points) Formulate appropriate null and alternative hypotheses to test the company's claim

H_0:	H_a:

(b) (10 points) Can the company claim that over 90% of the orders are delivered on time at the 2.5% level of significance? Choose an appropriate test statistic, calculate its value, describe the rejection rule and state whether we should reject or not the company's claim at the given level of significance.

Test statistic (formula):

Its value:

Rejection rule:

Decision (circle one): Do not reject company's claim Reject company's claim

(c) (7 points) Calculate the p-value and indicate at which levels of significance can the company make the claim that over 90% of the orders are delivered on time?

p-value:

Levels of significance: