MATH 218 FINAL EXAM December 17, 2008

Last Name: ________________________ First Name: __________________________

Student ID Number: _________________ Signature: __________________________

Circle your instructor's name and session time:

Bruck (11am), (12noon) He (10am) Lin (11am), (12noon) Lytvak (1pm), (2pm)

Mancera (10am), (2pm) Song (11am), (1pm) Yin (9am), (2pm)

INSTRUCTIONS

▪ Answer all ten problems. Numerical answers alone are not sufficient. **You must indicate how you derived them (show work) to obtain full credit**. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page.

▪ When an answer box is provided, put your final answer in the box.

▪ When submitting a numerical answer that is a decimal, use **four decimal places of accuracy** after the decimal point.

▪ When submitting a numerical answer that is a fraction, reduce it to lowest terms.

▪ Be sure to include the units in your answer.

▪ If you can not do part a) of some problem, but you need the answer for part b), then you can get partial credit for showing you know what to do. You could write, "let *p* be the answer to a)," and solve b) in terms of *p*.

▪ The value of a problem is indicated in parentheses following the problem number. The exam is worth a total of 200 points.

Problem	Value	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
Total	200	

Problem 1. (20 points) A purchasing department finds that 75% of its special orders are received on time. Of those orders that are on time, 80% meet specifications completely; of those orders that are late, 60% meet specifications.

a) (4 points) Construct a probability tree for this situation. Be sure to include the events, the probabilities, conditional probabilities, and joint probabilities, as appropriate.

b) (3 points) Convert the probability tree in part a) into a joint probability table. Be sure to include the marginal probabilities.

c) (3 points) Find the probability that a randomly selected order is on time and meets specifications.

c)

d) (3 points) Find the probability that a randomly selected order meets specifications.

d)

e) (4 points) If a randomly selected order doesn't meet the specifications, what is the probability that it is on time?

e)

f) (3 points) Suppose that 4 orders are selected at random and are independent of each other. Find the probability that all four orders meet specifications.

f)

Problem 2. (20 points) In a casino game, two special dice with only four faces each, labeled 1, 2, 3, 4 are tossed and the sum of the results is recorded. Let X denote this sum. Assume the dice are fair.

a) (6 points) Write down the probability distribution of X.

b) (8 points) Compute the mean and the standard deviation of X.

b) mean

b) standard deviation

c) (6 points) The casino will pay an entrant \$2 if the sum is between 6 and 7 inclusive, and \$8 if the sum is 8. Otherwise, it will not pay anything and the entrant loses the \$2 required to enter the game. What is the entrant's expected profit?

c)

Problem 3. (20 points) The joint probability distribution of two variables X and Y follows:

	Y	0	10	15	20	
X						
0		0.10	0.05	0.05	0.10	
5		0.10	0.15	0.05	0.10	
10		0	0.10	0.10	0.10	

a) (3 points) Label and insert the marginal probabilities in the table above.

b) (4 points) What is the probability that $X > 7$ given that $Y < 13$?

b)

c) (4 points) Are the variables X and Y independent? Please include your reasoning.

c)

d) (4 points) What is the probability that $X + Y < 21$?

d)

e) (5 points) What is the covariance between X and Y?

e)

Problem 4. (20 points) In a game show, there are 10 doors, behind which 2 contain a red sign, 3 have a yellow sign, and the rest have a blue sign. The contestant must choose 3 doors. If exactly 2 of the chosen doors have a red sign, the contestant wins a car. Otherwise, she wins nothing.

a) (5 points) What is the probability the contestant wins a car?

a)

b) (5 points) Now assume during the whole 50-minute episode of the game show, a total of 10 contestants will play the game. Find the probability that at least 1 of them wins a car.

b)

c) (4 points) Find the expected number of cars won per episode.

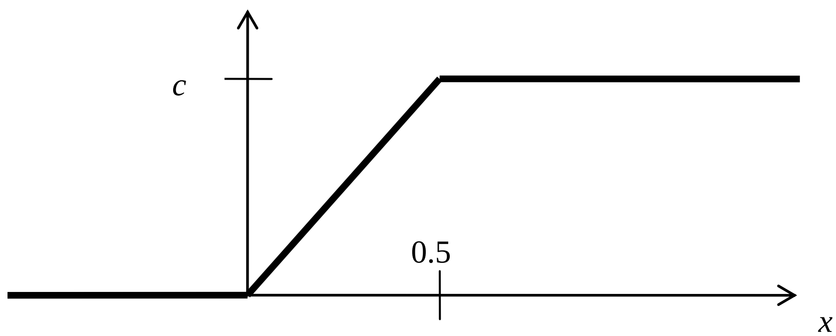
c)

d) (6 points) Now assume a season of the game show consists of 20 episodes. Find the probability that in each of at most 12 episodes at least one person wins a car. State whether your answer is exact or approximate, and explain.

d)

Problem 5. (20 points) Let X be a continuous random variable with the ***cumulative distribution function*** $F(x) = P(X \leq x)$. The graph of $F(x)$ is shown below, and $F(x)$ is given by

$$F(x) = \begin{cases} c, & \textit{if} \quad x > 0.5 \\ 2cx, & \textit{if} \quad 0 \leq x \leq 0.5 \\ 0, & \textit{if} \quad x < 0 \end{cases} \qquad (1)$$



a) (4 points) Find c.

a)

b) (4 points) Find the probability density function $f(x)$.

b)

c) (4 points) Find the expected value and the variance of X.

c)

d) (4 points) Find $P(X > \tfrac{1}{3} \mid X > \tfrac{1}{4})$.

d)

e) (4 points) Let X_1, X_2, X_3, X_4 be independent and identically distributed random variables with cumulative distribution function $F(x)$ as in (1) above, and let $\overline{X} = \dfrac{X_1 + X_2 + X_3 + X_4}{4}$.

Find $P(\overline{X} < \tfrac{1}{2})$.

e)

Problem 6. (20 points) The number of patients coming into a health clinic each hour is a Poisson distributed random variable with mean 10. For $i = 1, 2, 3, \cdots$, let X_i be the time in minutes between the arrival of the i^{th} patient and the arrival of the $(i+1)^{st}$ patient. Then, the $X_i's$ are independent and identically distributed exponential random variables. (Hint: Be careful with the units of time used.)

a) (6 points) What are $E[X_i]$ and $Var[X_i]$?

a) $E[X_i]$

a) $Var[X_i]$

b) (7 points) Let Y denote the average number of minutes that elapse between the arrival of successive patients beginning with the first and ending with the 31st patient, i.e., $Y = \frac{1}{30}\sum_{i=1}^{30} X_i$. Assume these patients form a random sample. What are $E[Y]$ and $Var[Y]$?

b) $E[Y]$

b) $Var[Y]$

c) (7 points) Estimate the probability that Y will be between 5 minutes 30 seconds and 7 minutes, inclusive.

c)

Problem 7. (20 points) A researcher reported annual incomes of $45,000, $35,000 and $55,000 for three new college graduates with math majors. Assume these graduates form a random sample. Assume that the annual incomes for recent math graduates are normally distributed.

a) (7 points) Estimate, with 95% confidence, the mean annual income for these recent math graduates.

a)

b) (7 points) Suppose we found that the population standard deviation of the annual incomes is $15,000. Find the new 95% confidence interval based on this additional information.

b)

c) (6 points) When we know that the population standard deviation is equal to $15,000 as in part b), how much larger of a sample does the researcher need if she wishes to estimate the mean annual income to within plus or minus $5,000 with 95% confidence?

c)

Problem 8. (20 points) A study conducted to estimate the proportion of businesses started in the year 2003 that have failed within five years of their startup, revealed that in a random sample of 40 of such businesses 11 of them had failed.

a) (7 points) Find a point estimate for the proportion p of all businesses that started in 2003 and have failed within 5 years.

a)

b) (7 points) Calculate a 90% confidence interval for p.

b)

c) (6 points) How large a sample size is needed in order to estimate p to within $\pm 5\%$ with 90% confidence? (i) Use a conservative value for $\hat{p}$ to obtain a sample size. (ii) Use the point estimate obtained in part a) to obtain a sample size. (iii) Compare your results. Which is larger?

c) (i) conservative:

c) (ii) using $\hat{p}$ from part a):

c) (iii) larger:

Problem 9. (20 points) A political pundit projects that in an upcoming vote a proposed constitutional amendment will pass with a supermajority, i.e., at least two-thirds of the votes. An opponent of the amendment doubts whether the amendment will actually pass with a supermajority. The opponent polls a random sample of 25 legislators and 13 support the constitutional amendment. Test the opponent's view. (Hint: Use care in setting up the alternative hypothesis.)

a) (4 points) Formulate the null and alternative hypotheses.

a) H_0:
a) H_a:

b) (2 points) State the alternative hypothesis in words.

c) (3 points) Define your sample value calculated from the data in words and give its numerical value.

c) Its value:

d) (4 points) Find the appropriate test statistic (with its formula) and calculate its value.

d) Test statistic (with formula):
d) Its value:

e) (3 points) Find the p-value.

e)

f) (4 points) At what levels of significance should the opponent conclude that the pundit's projection is wrong?

f)

Problem 10. (20 points) An investment management firm claims that on average a certain group of its mutual funds earn annual returns of at least 15%. A potential investor is skeptical and wishes to test whether the average of the annual returns is actually below 15%. A sample of 20 funds is examined. The annual returns of this sample are found to have a sample mean of 13% and sample standard deviation of 4%. Assume that the annual returns are normally distributed.

a) (5 points) Formulate the null and alternative hypotheses.

a) H_0:

a) H_a:

b) (5 points) Find the appropriate test statistic (with its formula) and calculate its value.

b) Test statistic (with formula):

b) Its value:

c) (5 points) At the 5% level of significance, find the rejection region.

c)

d) (5 points) What should the investor conclude regarding the average of the annual returns of these mutual funds? Explain.

d)