# MATH 218 FINAL EXAMINATION: FALL 2005

DIRECTIONS. On this examination, you may use a calculator and *one* 8-1/2 by 11-inch sheet of *handwritten* notes (both sides may be written on). No books or other notes are permitted.

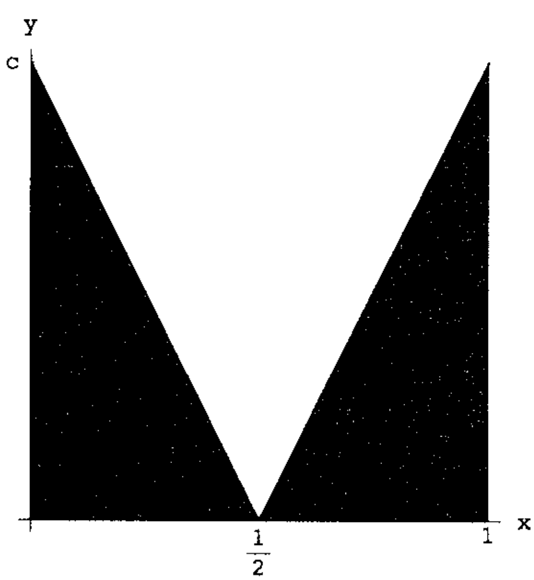
When an answer box is provided, copy your answer into that box. Numerical answers should be evaluated to be either decimals or fractions. **Numerical answers alone are not sufficient; you MUST indicate how you derived them (show your work).** When submitting a numerical answer which is a decimal, use the number of decimal places warranted by the data.

Some problems are worth more points than others; the value of a problem is indicated in parentheses following the problem number. The exam totals 200 points.

*Problem* 1 (20 pts). Matilda throws a big party to celebrate the $A$ she got in Math 218. Among many other things she prepared a very special fruit punch with fresh fruits from her garden. The punch is dispensed by a dispensing machine that she received as a gift when she successfully passed Math 118. The amount of fruit punch dispensed varies from cup to cup, and it may be looked upon as a random variable having a normal distribution with a mean of 7.7 oz and a standard deviation of 0.25 oz.

(a) Find the probability that the machine will dispense between 7.33 oz and 7.51 oz of fruit punch.
(b) Find the probability that the machine will dispense more than 8 oz.
(c) If 20 cups are filled with fruit punch, what is the expected number of cups that are filled with more than 8 oz?
(d) The machine has a knob which allows the mean to be reset, without changing the standard deviation. Matilda wants to reset the machine to make sure that 98.5% of the time it will not dispense more than 8 oz. What should the new mean be set to?

*Problem* 2 (20 pts). The graph of a probability density function $f(x)$ is shown below. The function has the properties that it is zero outside the interval $[0,1]$; that $f(1/2) = 0$ and $f(0) = f(1) = \mathsf{c}$, a constant to be determined; that it is linear on the interval $[0,1/2]$; that it is linear on $[1/2,1]$ with equation $y = 4x - 2$; and that it is continuous on $[0,1]$.




(a) What is the value of c, the height of the graph?
(b) Find the mean $\mu$ of a random variable which has $f(x)$ as its probability density function.

(c) Find the 95-th percentile for this distribution; i.e., if $X$ is a random variable with probabilit density function $f(x)$, find that value of $w$ such that

$$P(X \leq w) = 0.95.$$

*Problem* 3 (20 pts). A polygraph, or lie detector test, measures physical responses like blood pressure on test subjects in order to make a determination of whether a subject is lying or telling the truth. In a population of 10,000 subjects being screened for espionage, exactly 10 are spies, all of whom will lie during the polygraph. All the remaining individuals will tell the truth. The test can detect when a subject is lying 80% of the time, and will wrongly conclude that honest subjects are lying 15% of the time.

(a) Draw a tree diagram which represents this situation.

(b) What is the probability that an individual who fails the test is actually not a spy?

*Problem* 4 (25 pts). A mail-order business prides itself in its ability to fill customers' orders in six calender days, on average, the actual number of days being normally distributed. Recently doubts were raised about this claim by some angry customers, since their orders took more than six days to be filled. So the manager took a sample of 25 customers, found that the average number of days was 6.65, with a sample standard deviation of 1.5 days.

(a) Formulate the null and alternative hypotheses concerning the average number of days required.

(b) Is there enough evidence that the angry customers were right? Perform the test at the 5% significance level.

(c) If the mail-order business claim were true, the proportion of customers with more than six days delays should be 1/2; but the manager found that it took longer than six days for 18 of the 25 sampled customers. Use the data to check if there is a significantly higher proportion than expected. Use $\alpha = 5\%$.

*Problem* 5 (20 pts). The Elgin Heart Institute performs many open-heart surgery procedures. Recently research physicians at Elgin have developed a new heart bypass surgery procedure that they believe will reduce the average recovery time. The hospital will not recommend the new procedure unless there is substantial evidence to suggest that it is better than the existing procedure. Records indicate that the current mean recovery time for the standard procedure is 42 days, with a standard deviation of 5 days. To test whether the new procedure actually results in a lower mean recovery time, it was performed on a random sample of 36 patients. Assume the standard deviation remains 5 days.

(a) Formulate the null and alternative hypotheses.

(b) Assuming that the sample mean is 40.2 days, compute the test statistic.

(c) Conduct the test using a 5% level of significance and state clearly the conclusion.

(d) Find the probability that the sample mean from a typical sample of 36 recovery times would be as low or lower than 40.2, assuming the null hypothesis. What is this quantity called?

*Problem* 6 (20 pts). The annual rate of return (in percent), $X$, of stock A has the following probability distribution:

| $x$ | $-10$ | $10$ | $20$ |
|---|---|---|---|
| $P(X = x)$ | $1/4$ | $1/2$ | $1/4$ |

(For example, $-10$ represents the loss of 10% of the initial investment, while 20 represents a gain of 20% of the initial investment.) After one year, an investor will hold the amount of the initial investment plus any amount gained and minus any amount lost.

(a) Find the expected annual rate of return on stock A (in percent).

(b) Find the standard deviation of the annual rate of return on stock A (in percent).
(c) Jill starts with $1,000 and invests $500 in stock A and $500 in a risk-free asset at a constant annual rate of return of 3% (e.g., three-month U.S. Treasury bills). Find the expected value of Jill's investment (in dollars and cents) after one year.
(d) Find the standard deviation of Jill's investment (in dollars and cents) after one year.

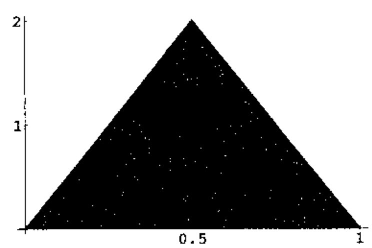
*Problem* 7 (20 pts). The times between consecutive customer arrivals at a particular bank teller's window are independent and exponentially distributed with a mean of 75 seconds.

(a) Find the probability that the time between consecutive customer arrivals at the window is less than 2 minutes.
(b) Find the average number of customers arriving at the window in a 5-minute period.
(c) Find the standard deviation of the number of customers arriving at the window in a 5-minute period.
(d) Find the probability that exactly 3 customers arrive at the window in a 5-minute period.

*Problem* 8 (25 pts). Alice Salem went trick-or-treating on Halloween and collected from her neighbors 15 chocolates, 10 cookies and 20 hard candies.

(a) Every day she eats 6 of her treats at random. What is the probability that the first day she picks 3 hard candies among her 6?
(b) Alice eats 2 treats at random every morning and 4 treats at random every evening. What is the probability that on the first day exactly 3 of the 4 treats she picks in the evening are hard candies, given that she ate 2 chocolates that morning?
(c) At the end of each evening Alice's father replaces exactly the treats that Alice consumed that day (thereby keeping constant the number of chocolates, the number of cookies, and the number of hard candies). Alice has discovered that she likes chocolate in the morning, and so every morning she eats 2. What is the probability that on at least 15 of the next 50 days, she picks 3 hard candies in the evening (as in part (b))?

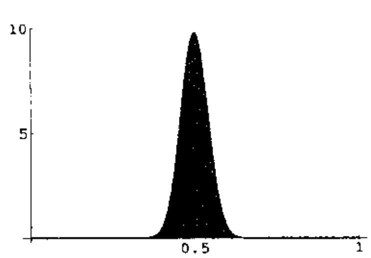
*Problem* 9 (10 pts). A random variable $X$ has probability density function as given below:



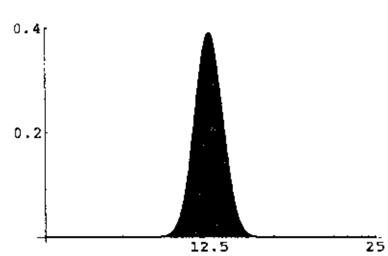

25 independent random variables $X_1, X_2, X_3, \ldots, X_{25}$ with this density function are averaged, to yield

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \cdots + X_{25}}{25}.$$

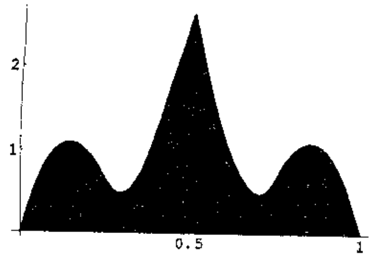
Which of the following graphs most closely resembles the probability density function for $\overline{X}$?



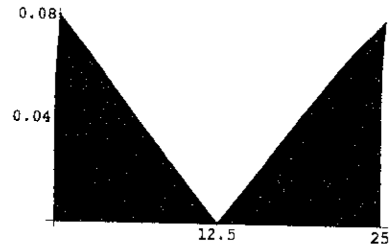

(a)




(b)




(c)




(d)

**Explain your reasoning:**

*Problem* 10 (20 pts). Assume that the yield per acre for a particular variety of soybeans is normally distributed. For a random sample of 5 plots, the yields in bushels per acre were 37.4, 48.8, 46.9, 55.0, and 44.0.

(a) Find the sample mean.
(b) Find the sample standard deviation.
(c) Find the 90% confidence interval (CI) for the population mean.
(d) Is the 95% CI wider or narrower than the 90% CI, and why?