MATH 218 **FINAL EXAM** **Fall 2004**

December 15, 2004

INSTRUCTIONS.

- **Show All Work.** No credit in general for answers only, if there is work to be shown. Simplify answers to a fraction or decimal, and write this final answer in the box given, if one is provided. Keep at least 3 digits of accuracy after the decimal point. Use the back of the answer sheet for more space.
- If you can't do part (a) of some problem, but you need the answer for (b), then you can get partial credit for showing you know what to do. You could write, "Let p be the answer to (a)," and solve (b) in terms of p.
- A calculator is allowed but it must not have any additional capabilities: no cell phones, Palm Pilots, etc. that can function as calculators.

Problem 1. *A survey of those households which have two registered voters found the following joint distribution for the number X of votes cast for Bush, and the number Y cast for Kerry, in the household:*

		Y			
		0	1	2	
X	0	.05	.05	.35	
	1	.05	.10	0	
	2	.40	0	0	

(a) *What is the probability, among these households, that both registered voters actually voted?*
(b) *Are X and Y independent? State how you know.*
(c) *Make a table of the marginal distribution of X.*
(d) *Find the expected value of X.*

Problem 2. *At an art exhibition there are 12 paintings of which 10 are authentic originals. A visitor selects a painting at random and before she decides to buy, she asks the opinion of an expert about the authenticity of the painting. The expert marks the painting REAL (if he believe it is authentic) or FAKE (if he believes it is a copy.) He is right in 90% of cases on average, both with authentic paintings and non-authentic copies.*

(a) *Make a probability tree for this situation.*
(b) *Given that the expert marks a painting FAKE, what is the probability that it is authentic, and what is the probability it is a copy?*
(c) *If the expert (possibly incorrectly) marks the painting FAKE, then the visitor returns the painting and chooses another one of the remaining 11 paintings. What is then the probability that her second choice is an authentic original?*

Problem 3. *Suppose that a Math 218 discussion section has 24 students, including 18 male and 6 female. 7 students receive a grade of B.*

(a) *What is the probability that exactly 2 female students in a discussion session got a B as overall grade? (This answer is complicated to simplify so you can skip doing so. For use below, the numerical value is .371.)*
(b) *In a particular semester there are 36 discussion sections, each as above with 18 males, 6 females and 7 B grades. If Y is the number of discussion sessions with exactly 2 female students with final grade B (as in part (a)), what is the distribution of Y? Specify the name (Poisson, normal, exponential, binomial, etc.) and parameters.*
(c) *What is the approximate probability that at most 10 of the discussion sessions have exactly 2 female students with grade B?*

Problem 4. *The amount of time X, in minutes, that a guest at Sweet-Dreems Hotel spends waiting for an elevator is described by the following probability density function:*

$$f(x) = 12(x^3 - 2x^2 + x), \quad 0 \le x \le 1$$

a) *Find the mean amount of time a guest spends waiting for an elevator.*
b) *Find the probability that a guest waits less than 0.5 minute for an elevator.*
c) *Mary, a guest at the hotel, used the elevator 6 different times yesterday. Find the probability that she waited more than 0.5 minute exactly two of these times.*

Problem 5. *Acme Consolidated Products sells extreme-sports bikes, and plaster casts for broken bones. Let X be the number of extreme-sports bikes sold in a week, and Y the number of plaster casts sold in a week. Suppose $E(X) = 10, E(Y) = 12$, $var(X) = 9$, $var(Y) = 16$. Acme finds that the extreme-sports bike sales X and plaster cast sales Y have a positive correlation of 0.4.*

(a) *Find the mean and standard deviation of the total number of items (extreme-sports bikes plus plaster casts) sold in a week.*
(b) *Suppose the profit per extreme-sports bike (in hundreds of dollars) is 5, and the profit per cast is 2. Find the mean and standard deviation of the total profit on extreme-sports bikes plus plaster casts.*

Problem 6. *In accordance with the opinion of some seismologists we shall assume that earthquakes on a particular section of the San Andreas fault form a Poisson experiment, with an average rate of 1 per 78 years.*

(a) *Given that on that section of the San Andreas fault there has been no earthquake for the last 30 years, what is the probability that the next one will occur within the next 50 years?*
(b) *What is the probability that at least 2 earthquakes struck since Juan Bautista Anza travelled through Southern California in 1775?*

Problem 7. *The weekly output of a steel mill is a uniformly distributed random variable that lies between* 110 *and* 175 *metric tons.*

(a) *The mill owners want to know what output places a given week in the bottom 25% of weekly outputs. How low must the output be, to fall in the bottom 25%?*
(b) *Suppose the weekly outputs are independent of each other and that a year has* 52 *weeks. Find the probability that total output of the steel mill in a year will be less than 7500 metric tons. State whether your answer is exact or approximate.*

Problem 8. *In a random sample of 9 senior executives of US companies, the mean salary bonus during the last tax year (in millions of dollars) is 0.623, with the sample standard deviation of 0.187. Assume salaries are normally distributed.*

(a) *What is the 90% confidence interval for the mean salary bonus in the population of senior US executives in the last year?*
(b) *Assume now (unrealistically) that we know that the population standard deviation σ is also equal to 0.187. What is the 90% confidence interval under this assumption?*
(c) *What sample size do we need in order to cut in half the width of the confidence interval in part (b)?*

Problem 9. *An agent for an international relief organization suspects that the most recent shipment of flour bags (which are supposed to weigh 50 kg each) might amount to less than is claimed. The agent takes a sample of 25 bags and determines their weights. It is found that the sample mean $\overline{X} = 49$ kg and the sample standard deviation $s = 2.5$ kg. Assume that the population of flour bags is normally distributed.*

(a) *Formulate the null and alternative hypotheses.*
(b) *Determine the p-value of the test result. (Give an exact answer if that is possible with the tables available; otherwise give an approximate answer.)*
(c) *Determine at which of the following significance levels H_0 should be rejected (circle all that apply):*
10% 7% 5% 2.5% 1%.
(d) *We now change one assumption: 2.5 kg is now a known standard deviation, instead of a sample standard deviation. Suppose the population mean is actually 48.5 kg, instead of the claimed 50 kg. State*
  (i) *the rejection region for test statistic $\overline{X}$ for a test at the 5% level;*
  (ii) *what kind of error (Type I or II) is made if we fail to reject H_0;*
  (iii) *what the probability is for such an error.*

Problem 10. *A forest management company is interested in the survival of planted trees in a region. They would like to know whether the proportion surviving 1 year or more differs from usual standard of* 0.25. *The scientists plant 400 trees distributed over a wide area.*

(a) *Formulate the null hypothesis and the alternative hypothesis.*
(b) *Select a test statistic and define the rejection region with a significance level of 10%.*
(c) *In a particular survey, the number of surviving trees after one year is 116 out of 400. Find the p-value of the result and determine whether or not there is sufficient information to conclude that the proportion surviving differs from the usual standard of 0.25.*