MATH 218 FINAL EXAMINATION

December 17, 2003

Professors: J. Colwell, F. Lin, K. Styrkas, E. Verona, Z. Vorel.

Problem 1. A random sample of 50 purchases at a department store produced the following contingency table for the method of payment and the size of the purchase:

	Cash	Credit card	Debit card
Under \$30	5	2	2
\$30 to \$100	2	10	9
Over \$100	1	12	7

(a) Given that a purchase was made with a debit card, find the probability that it was at least \$30.

(b) Given that a credit card purchase was at least \$30, find the probability that it was over \$100.

(c) Find the probability that a purchase was under \$30.

(d) Find the probability that a purchase was paid by cash.

(e) Are "Under \$30" and "Cash" independent events? Explain.

Problem 2. A carnival has three games. In Game A, the player has a 4% chance of winning. In Game B, the player has a 3% chance, and in Game C, a 2% chance. You know that your friend, who likes the carnival, is equally likely to play any of the three games.

(a) Draw a tree diagram to depict this situation. Include all events and probabilities (conditional and joint) involved.

(b) You go to the carnival and see your friend with a stuffed dragon which she won at one of the games. What is the probability that she won it at Game A?

(c) Inspired by your friend's success, you decide to play one game. Unfortunately, Game A has ceased to operate for the day. In order to decide whether to play Game B or Game C, you first flip a fair coin. What is the probability that you win a game?

Problem 3. An investor's portfolio will be made up of two financial instruments – stocks and bonds.

(a) There are 9 units of stocks and 6 units of bonds available. Suppose an investor selects 10 units of securities from these stocks and bonds at random *without replacement* to make up a portfolio. Find the probability that the portfolio will contain 6 units of stocks and 4 units of bonds.

(b) For a particular stock, the probability that its price increases from its price on the previous trading day is 0.6. Find the probability that its price increases on exactly 6 of the next 10 trading days.

(c) Consider the same stock as in part (b). Find the probability that its price increases on between 50 and 70 (inclusive) of the next 100 trading days.

Problem 4. Travelling along the Icefields Parkway in the Rocky Mountains, motorists sometimes see mountain goats standing at the side of the road. The points along the highway where the animals are sighted are random and independent. Driving along any 10km stretch of the highway, the probability that a motorist will pass exactly k mountain goats is

$$\frac{e^{-2.5} \cdot (2.5)^k}{k!}.$$

(a) What is the mean distance along the highway a motorist must travel before passing the first mountain goat?

(b) What is the standard deviation of the distance along the highway a motorist travels between consecutive sightings of mountain goats?

(c) You are driving along the highway and have seen no mountain goats in the last 6km. What is the probability that you pass the next mountain goat between 1km and 3km down the road?

(d) What is the probability that you pass exactly 2 mountain goats in the next 5km?

Problem 5. Suppose that the probability density function of a random variable X is

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & \text{if } -1 \le x \le 0 \\ -\frac{x}{6} + \frac{1}{2}, & \text{if } \quad 0 \le x \le 3 \\ 0 & \quad \text{elsewhere} \end{cases}$$

(a) Find $P(-\frac{1}{2} < X < \frac{1}{2})$.

(b) Find the number c such that $P(X < c) = \frac{1}{16}$.

(c) Given that the expected value of X is equal to $2/3$, find the variance of X.

Problem 6. (a) Let U be a normally distributed random variable with mean 10 and variance 4. Find $P(|U - 9| > 3)$.

(b) Assume Y is normally distributed with $E(Y) = 20$ and $P(18 < Y < 22) = 0.34$. Find the standard deviation.

(c) X is a normal random variable with mean 15 and standard deviation 1.5. Let $\overline{X}$ be the mean of a random sample of size 4. Find $P(14 < \overline{X} < 17)$.

Problem 7. In a big city, let X denote the number of movie rentals per household in a week, and Y denote the number of video game rentals per household in a week. The joint distribution of X (movie rentals) and Y (video game rentals) is given below.

		X			
		0	1	2	3
Y	0	0.02	0.03	0.04	0.01
	1	0.06	0.15	0.25	0.14
	2	0.02	0.12	0.11	0.05

(a) Find the exact probability that the total number of rentals (movies and games) of a randomly chosen household in a week is greater than three.

(b) Find the marginal probability distribution of X.

(c) Find the expected value, and standard deviation of X.

(d) 100 households are randomly selected. Find the probability that the total number of **movies** rented in a week by these households is 160 or fewer.

Problem 8. A survey of 50 people at a movie theater showed that 38 of them bought popcorn at the theater.

(a) Find the point estimate for the proportion p of people who buy popcorn at the movies.

(b) Calculate the 90% confidence interval for p.

(c) You want to conduct another survey to estimate p to within 5% with 90% confidence. How large should your sample size be?

Problem 9. The Healthy Food Company claims that its cereal boxes contain, on average, 453 grams of cereal. We suspect that the cereal boxes contain, on average, less than claimed. You decide to test the claim by inspecting 6 randomly selected boxes, and get the following weights:

$$454, \quad 447, \quad 452, \quad 446, \quad 450, \quad 445.$$

Assume that the amount of cereal in a box follows a normal distribution.

(a) Formulate the null and the alternative hypotheses.

(b) Which statistic should be used to test these hypotheses? Evaluate it numerically.

(c) Formulate the rejection rule at 5% significance level. Using your computation in (b), decide whether the null hypothesis should be rejected.

(d) At which of the following significance level(s) should the null hypothesis be rejected? Circle all that apply.

0.005 0.01 0.025 0.05 0.1

Problem 10. A survey of the morning beverage market shows that the primary breakfast beverage for 17% of Americans is milk. A milk producer in Wisconsin, where milk is plentiful, believes the figure is higher for Wisconsin. To test this idea, she contacts a random sample of 550 Wisconsin residents and asks which primary beverage they consumed for breakfast that day. Suppose 115 replied that milk was their primary beverage. The null and alternative hypotheses for her test are:

$$H_0\colon p = 0.17 \qquad \text{and} \qquad H_1 : p > 0.17.$$

(a) Which test statistic should be used to test these hypotheses? Evaluate its numerical value. All computations should be done with at least 5 decimals (5 digits after the decimal point).

(b) Find the P-value.

(c) Based on the P-value, at which significance level(s) should she reject the null hypothesis? Circle all that apply.

0.3 %　　0.5%　　2.5%　　3%