Fall 2002 Math 218 Final Examination

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Problem 1. In a large population, 1% of individuals are infected with a certain virus, say V. A test is applied to the whole population, and it is assumed that 99% of infected individuals test positive while 2% of noninfected individuals test positive. Let A be the event that a randomly chosen individual is infected with the virus V, and let B be the event that a randomly chosen individual tests positive for V.

a) Construct a probability tree diagram describing this situation.
b) Are events A and B independent? (Show your work.)
c) For events A and B in the previous problem, find $P(A|B)$ and $P(\overline{A}|B)$.
d) Two randomly chosen individuals that are not infected with V are going to take this test. Find the probability that both of them will test positive for V.

Problem 2. A box contains 5 quarters, 4 dimes, and 2 nickels.

a) We select 8 coins randomly without replacement. Find the probability that 4 or more of the selected coins are quarters.
b) A coin is drawn randomly 5 times with replacement. Find the probability that we get at least one quarter.
c) A coin is drawn 100 times with replacement. Find the probability that a quarter appears 50 or more times.

Problem 3. Customer arrivals to the Last-Minute-Holiday-Shopping-Store follow a Poisson distribution with the rate of 2 per hour.

a) Find the probability that there will be 2 or more customers between 9:30AM and 11:30AM?
b) Find the probability that there will be no customers between 10:30am and 11:00am?
c) A customer arrives at noon. What is the expected amount of time that the store clerk has to wait until the next customer arrives?
d) Another customer arrives at 1:00pm. Find the probability that the next customer arrives before 1:45pm.

Problem 4. The time, in minutes, required by a certain assembly operation, is a random variable T with the following probability density function:

$$f(t) = \begin{cases} \frac{3}{8}t^2 & 0 < t < 2, \\ 0 & \text{otherwise.} \end{cases}$$

a) Find the the probability that the operation is complete in 1 minute or less.
b) The operation is not complete after 1 minute. What is the probability it will be complete in the next 0.5 minutes?
c) Find the expected value of T.
d) The standard deviation of T is $\sqrt{3/20}$ minutes. What is an approximate probability that 240 consecutive operations of this kind are finished in 6 hours 9 minutes or less? Assume that the operations are performed independently of each other, and the next operations starts immediately after the previous one is finished.

Problem 5. Lengths of sardines processed by a cannery are normally distributed with a mean of 4.54 inches and a standard deviation of 0.25 inch.

a) What percentage of the sardines are shorter than 4.00 inches?
b) Exactly 85% of the sardines are shorter than c inches. Find c.
c) If a sample of 25 sardines is selected at random, what is the probability that their mean length exceeds 4.65 inches?

Problem 6. In a large community, the number of children X under 18 per household is a random variable with the following distribution function.

c	0	1	2	3
$P(X=c)$	0.1	0.3	0.4	0.2

a) What is the probability that a household has at least one child under 18?
b) Find the expected value and standard deviation of X.
c) 100 households are randomly selected. Find the probability that the total number of children in these households is 160 or fewer.
d) Two households are randomly selected. Find the probability that the total number of children in these households is 5 or more.

Problem 7. A study based on a random sample of 40 mutual funds revealed that 12 of them had annual fees of 1.2%.

a) Find a point estimate for the proportion p of all mutual funds that charge annual fees of 1.2%.
b) Calculate the 90% confidence interval $[L, R]$ for p.
c) Which of the following is the best interpretation of the interval $[L, R]$ you constructed in (b) (Circle all that apply):
   (i) Consider 1000 samples of mutual funds and their similarly constructed intervals; the proportion of all mutual funds that charge annual fees of 1.2% lies in about 900 of these intervals.
   (ii) The amount of annual fees charged by 90% of the mutual funds in the sample is between L and R.
   (iii) If we take another sample of mutual funds, there is a 90% chance that the proportion of them which charge annual fees of 1.2% is between L and R.
d) How much larger should the sample be in order to estimate p to within $\pm 5\%$ with 90% confidence.

Problem 8. The Thin Line Company claims that the individuals using its weight loss product for a duration of three months lose at least 10% of their total body weight, on average. In what follows, you can assume normal distribution for the weight loss percentage.

a) A consumer protection agency suspects that the company's claim is false. Formulate the agency's null and alternative hypotheses.
b) The agency tests the product on a random sample of 16 individuals, and finds the average body weight loss to be 9%, with a sample standard deviation of 4%. Compute the test statistic value.
c) Is there enough evidence that the company's claim is false, at the 5% significance level?
d) Estimate the P-value. Circle one, *and explain*.
   (1) The P-value is less than 0.01.
   (2) The P-value is between 0.01 and 0.025.
   (3) The P-value is between 0.025 and 0.05.
   (4) The P-value is between 0.05 and 0.1.
   (5) The P-value is greater than 0.1.

Problem 9. A UNICEF study in 1995 showed that 31% of all children in country X suffered chronic malnutrition. A researcher claims that this percentage is even higher now. She plans to do a test of hypothesis to gather evidence to support her claim. The null and alternative hypotheses of her test are $H_0 : p \leq 0.31$ and $H_a : p > 0.31$.

a) Which of the following best describes the p that appears in the null and alternative hypotheses? Circle your answer.
   i) The fraction of children in country X in 1995 that suffered chronic malnutrition.
   ii) The fraction of children in country X today that suffer chronic malnutrition.
   iii) The fraction of children in country X in the sample that suffer chronic malnutrition.
b) In a random sample of 207 children of country X, 79 suffered chronic malnutrition. Find the P-value.
c) At which of the following significance levels can the researcher reject the null hypothesis? (Circle all that apply.)
   1% 2% 5% 10%