Fall 2001 Math 218 common final exam

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Problem 1 (20 points). In a MATH 218 class there are a total of 50 students. There are 30 female students, 9 of whom do not own a car. Among the male students 6 do not own a car.

(a) Use *either* a contingency table *or* a tree, to organize the data.
(b) Find the probability that a randomly selected student owns a car and is female.
(c) Find the probability that a randomly selected student is female, given that the student owns a car.
(d) Are the events "is female" and "owns a car" independent? Explain.

Problem 2 (15 points). The probability of at least one Category 5 (C5) hurricane occurring in the Pacific Ocean during any given year is 10%. Scientists at the Ocean Institute have been predicting whether or not a C5 hurricane would occur in each calendar year with mixed results. In years when C5 hurricanes have occurred, their predictions have been correct 60% of the time. In years when no C5 hurricane occurred they have been correct 80% of the time.

a) The Ocean Institute is predicting that a C5 hurricane will occur in 2002. Find the probability that such an event will occur given their previous record.
b) Scientists at the rival Atmospheric Sciences Service predict a C5 event in 2003 and have predicted such an event every year. Find the probability of a C5 event based on the Atmospheric Sciences Service's record.

Problem 3 (20 points). The Giant Miniatures Company is implementing the Ultimate Quality Review (UQR) staff development program by randomly choosing managers for training. Among all the managers in the company, 35% have already completed the older Total Program Improvement (TPI) training program.

a) If an initial group of 10 managers is chosen at random from the company, find the probability that at least 2 of them will have completed the TPI training.
b) A total of 120 managers from Giant Miniatures will be trained in the UQR program. Approximate the probability that between 30 and 40, inclusive, of these randomly selected managers will be graduates of the TPI program.
c) The Tiny Titan Company is also implementing the UQR program. There are a total of 19 managers in the company, and 12 of them are graduates of the TPI program. If 5 managers are chosen at random from those 19 managers, then find the probability that exactly 3 of them are graduates of the TPI program.

Problem 4 (15 points). The number of seismic events occurring along a certain stretch of the San Andreas Fault is Poisson-distributed with a mean rate of 0.5 events per day.

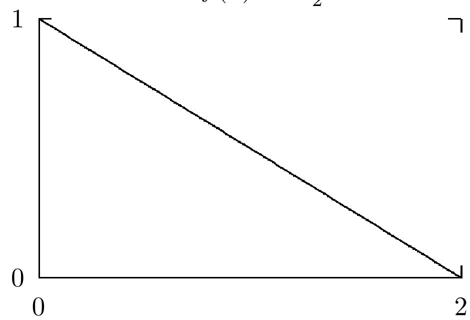
a) What is the expected number of earthquakes along this stretch of the San Andreas Fault in the next week?
b) Compute the probability that there will be at least 3 seismic events along this stretch of the San Andreas Fault in the next week.
c) Let T be the number of days between consecutive seismic events. It can be shown that T is an exponentially distributed random variable with a mean of 2 days. Compute the probability that the time between earthquakes is between 1 and 3 days.

Problem 5 (15 points). A manager responsible for the maintenance of widgets is setting up a schedule for checking and replacing worn widgets. The probability density function for the lifetime (in years) of this type of widget is

$$f(x) = -\frac{1}{2}x + 1 \quad \text{for} \quad 0 \le x \le 2.$$

a) Find the probability that a randomly selected widget is still working properly when it is 6 months old.
b) Given that a randomly selected widget *is* working properly when it is 6 months old, find the probability that it will *still* be in working order when it is 16 months old.

Plot of $f(x) = -\frac{1}{2}x + 1$



c) Find the expected lifetime (in years) for this type of widget.

Problem 6 (20 points). At Crystal Creek Trout Farm, the trout have lengths that are normally distributed with a mean of 13 inches and a standard deviation of 4 inches. Any trout that are caught whose lengths are less than 8 inches must be returned to the creek.

a) Assuming that all fish are equally likely to be caught, what proportion of fish that are caught have to be returned to the creek?
b) If you catch 3 fish, what's the probability that you have to return at least 1 of them?
c) Managers at Crystal Creek are concerned that the creek is being over-fished. They propose increasing the minimum length of fish that are retained. If they want 90% of caught fish to be returned to the creek, what should they set this minimum length to be?

Problem 7 (15 points). A portfolio manager has five mutual funds that she invests in. At the end of every month, the monthly returns of the funds are compared to the monthly returns of the S&P500 market index. If the return of a fund exceeds the return of the S&P500, then she gets a reward. Let Y be the number of rewards that she receives in a month. The probability distribution of Y is

Y	0	1	2	3	4	5
Probability	0.25	0.20	0.18	0.14	0.13	0.10

The monthly performances of funds are independent of each other and from month to month.

a) What is the expected number of rewards that she receives in a month?
b) What is the standard deviation of the number of rewards that she receives in a month?
c) Find the approximate probability that the total number of rewards that she receives over the next 3 years is at least 60.

Problem 8 (20 points). A recent study of drivers in the Stop'n'Go driving school involved 5 randomly selected drivers. Asked to specify how many hours it took them to learn how to drive, their responses were

23 24 30 25 23

Assume that learning times for all drivers in this school are normally distributed.

a) Find the 95% confidence interval for the mean learning time of drivers in this school.
b) Suppose we discover that the standard deviation of learning times in this school is 2 hours. Find the new 95% confidence interval based on this additional information.

Problem 9 (20 points). Friendly Medicine, Inc., a pharmaceutical company, has developed a new sleeping pill and has decided to test it on a group of 50 volunteers. It turns out that 7 of them developed a sleeping disorder after taking the pill.

a) Next year the pill is going to appear on the market. Find the 90% confidence interval for the proportion of users who develop a sleeping disorder after taking the pill.
b) How many additional volunteers does the company need in order to estimate this proportion to within ±2% with 90% confidence?

Problem 10 (20 points). The Brown Kow Dairy sells containers of milk. These containers are stamped with an expiration date; those containers not sold by the expiration date are discarded. Brown Kow claims that on average, milk can be kept for at least 14 days past the expiration date before going bad. To test this claim, government inspectors decided to keep 30 containers and record how long past its expiration date it takes each container to go bad.

a) Formulate the null and alternative hypotheses H_0 and H_a for this test.
b) Choose a test statistic for the test, and give a rejection rule at the 5% level of significance.
c) The inspectors find that the average time it took the 30 containers to go bad was 13 days, with a sample standard deviation $s = 2.1$ days. What is the inspectors' conclusion? Show your work.

Decision (circle one): Not Reject H_0 Reject H_0

d) What can you say about the P-value for this test (circle one)? Show your work.
   (1) The P-value is less than 0.01.
   (2) The P-value is between 0.01 and 0.025.
   (3) The P-value is between 0.025 and 0.05.
   (4) The P-value is between 0.05 and 0.1.
   (5) The P-value is greater than 0.1.

Problem 11 (20 points). Santa Claus divides the worlds children into two distinct groups; naughty and nice. Historically, he has found that 20% of the world's children are naughty. However, last year he suspected that this proportion may have changed.

In an effort to avoid loading either too many or too few presents on his sleigh on Christmas Eve, Santa decided to use sampling and hypothesis testing. See if there is sufficient evidence to support his suspicion.

a) Formulate an appropriate null and alternative hypothesis for Santa to use, and enter them in the boxes:
b) Choose an appropriate test statistic and find a rejection rule at the 5% level of significance.
c) Last December, Santa sampled households with a total of 150 children in the 90210 zip code (Beverly Hills). In this group of 150 children, 40 were naughty. Compute the value of the test statistic in part b.
d) Based on the value of the test statistic found in part c, and the rejection rule given in part b, Santa decided to either reject or not reject the null hypothesis at the 5% level of significance. What did Santa decide?

Decision (circle one): Not Reject H_0 Reject H_0

e) As it turned out, last year Santa did not bring enough toys with him, leaving him somewhat skeptical of statistical analysis. This year, if Santa were to hire you as a consultant, which (if any) of the following would you advise him to do? Circle all that apply

   a. Change the level of significance to 7.5%.

   b. Take more data in Beverly Hills.

   c. Try to insure that his sample is random.

   d. Take Math 218 in the spring.