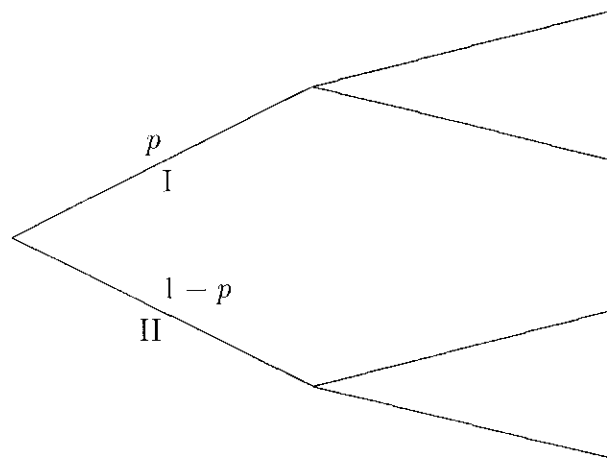


Directions

Every numerical answer should be simplified to a fraction or a decimal, unless otherwise stated. You must show your work and justify your methods to obtain full credit. Use the continuity correction wherever it is appropriate unless otherwise instructed. If you can't do one part of a problem but need that answer later, guess an answer and use that guess for the later part. The exam is worth a total of 200 points.

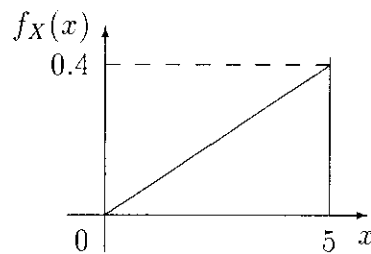
1. (20 points) A company staff consists of 465 women and 135 men. The annual salaries of 120 of the women and 45 of the men are higher than \$40,000. You are the analyst studying the problem of gender pay discrimination.
 - a) (5 points) Draw the contingency table for this case.
 - b) (5 points) One of the men is selected at random. What is the probability that he is a lower paid employee (salary at most \$40,000)?
 - c) (5 points) One of the staff is selected at random. Are the events "being a man" and "being higher paid" independent in this company? Justify your answer with the appropriate calculation.
 - d) (5 points) What is the probability that a randomly selected staff member is either a woman or higher paid individual?

2. (15 points) 2% of all Toyotas bought at a certain dealership break down during their first year of ownership. The dealership has two suppliers, I and II. 3.5% of all cars produced by supplier I have defects which would cause breakdowns during the first year of ownership, while 99% of all cars produced by supplier II never break down during the first year.
 - a) (7 points) A car is selected at random from the dealership stock. Find the probability p that the selected car is produced by supplier I. *Hint: Fill in the following tree diagram and use it to help you write down an equation to solve for p .*

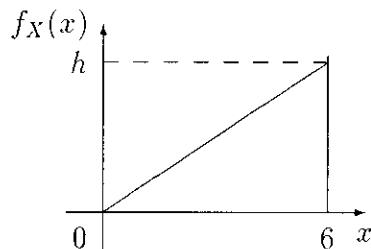


- b) (8 points) Your Toyota, bought at this dealership, has just broken down after five months of operation. What is the probability that it was produced by supplier I? Use $p = 0.3$ if you were not able to solve part a).

3. (20 points) A small consulting firm has 15 consultants: 9 in the Los Angeles office and 6 in the San Diego office.
- (6 points) How many ways can a president, vice president, and secretary be chosen from the 9 consultants in the Los Angeles office?
 - (7 points) Four consultants are needed to work on a project for the EPA. How many ways can these four consultants be chosen from among the fifteen consultants?
 - (7 points) If four consultants are chosen at random, what is the probability that at most 2 of them are from the Los Angeles office?
4. (15 points) At a particular cafeteria, the manager has found that at breakfast time, 60% of the customers buy a cup of coffee priced at \$1.25. Assume that the next 10 customers form a random sample.
- (5 points) Find the probability that 8 or more of these 10 customers will buy coffee.
 - (5 points) Find the expected revenue on coffee generated by these 10 customers.
 - (5 points) Find the standard deviation in the revenue on coffee generated by these 10 customers.
5. (20 points) Let X be the random variable describing the time between route A bus arrivals at a particular bus stop. The probability density function of X is as follows.



- (7 points) Determine $P(X > 3)$.
- (7 points) Determine c so that $P(X > c) = 0.20$.
- (6 points) Suppose now that the random variable X is the time between route B bus arrivals, which assumes values between 0 and 6 minutes, as shown in the following figure. What is the value for h ?



6. (20 points) The length of calls to telephone operators in the customer service division of Amazing.com is normally distributed with a mean of 12 minutes and a standard deviation of 3 minutes. For a quality control study the executives of Amazing.com sample 100 calls (you may consider the 100 calls to be a random sample).
- (6 points) What is the expected value and standard deviation for the sample mean?
 - (7 points) What is the probability that the sample mean is at most 12.5 minutes?

- c) (7 points) One afternoon one of the operators handles 44 phone calls. What is the probability that this operator works over time (i.e. more than 8 hours, or 480 minutes)?
7. (15 points) A red ball, a green ball, and a white ball are placed in a bin to award monetary prizes to the winners of a weekly contest. The red ball is worth \$300, the green ball is worth \$200, and the white ball is worth \$100.
- a) (5 points) The winner of the first week's contest picks a ball from the bin at random and returns it to the bin. Let X be the amount awarded to this first winner. What is the mean μ and the standard deviation σ of X ?
- b) (5 points) The winner of the second week's contest picks a ball from the bin at random and returns it to the bin. Calculate the probability that the average amount of the two prizes awarded will be over \$245.
- c) (5 points) If the contest continues for 35 weeks, calculate the approximate probability that the average of the 35 prizes awarded exceeds \$245. (If you were unable to answer part a, use $\mu = 225$ and $\sigma = 74.9$.)

8. (15 points) A sample of size three from a normal population with mean μ and variance σ^2 yields the values

$$X_1 = 7.8, \quad X_2 = 10.2, \quad \text{and} \quad X_3 = 10.6.$$

- a) (5 points) Compute the (usual) point estimates for μ and σ .
- b) (10 points) Construct a 98% confidence interval for the unknown population mean.
9. (20 points) A pollster takes a random sample of 100 voters in Orange County and finds that 24% are in favor of Prop A.
- a) (8 points) Construct a 90% confidence interval for the fraction of voters in Orange County who are in favor of Prop A.
- b) (12 points) Based on the preliminary sample, what sample size would be required to construct a 99% confidence interval of width 2%?
10. (20 points) The height of American adult males is normally distributed with a mean of 69.1 inches. A new drug to combat math anxiety is discovered but it is suspected that when used by children it may affect the child's growth. The FDA commissions a long term study to ascertain if there is any evidence for this assertion.
- a) (5 points) Formulate the null and alternative hypotheses and choose an appropriate test statistic.
- b) (5 points) In the FDA study 20 children that took the drug regularly between the ages of 8 and 10 years are monitored until they are adults. It is found that their height as adults has an average of 70.0 inches and a sample standard deviation of 2.1 inches. Calculate the value of the test statistic. Then estimate the P -value and justify your answer.
- P -value is (circle one): (i) less than 0.005 (ii) 0.005 to 0.01 (iii) 0.01 to 0.025 (iv) 0.025 to 0.05 (v) 0.05 to 0.10 (vi) 0.10 to 0.20 (vii) more than 0.20
- c) (5 points) At which of the following significance levels is it appropriate to claim evidence for the alternative hypothesis? (Circle all that apply.) 1%, 2%, 5%, 10%

d) (5 points) Researchers at the NIH do a similar but much larger study. They also get an average height of 70.0 inches and sample standard deviation of 2.1 inches. Is their P -value

(i) less than,

(ii) equal to, or

(iii) greater than

the FDA's P -value? Circle one and explain.

11. (20 points) According to the Census Bureau, at least 73.2% of workers drive alone in their cars to work. A city recently implemented a car pool program with the ultimate goal of reducing this percentage. Three months after implementation of the program the city commissioned a survey to assess its effectiveness. The survey selected 100 workers at random and collected information on how each worker got to work.

a) (6 points) Formulate the null and alternative hypotheses and choose an appropriate test statistic.

b) (8 points) Find the rejection region if the test is to be performed at the 5% significance level.

c) (6 points) The data collected in the survey found that 68 of the 100 workers sampled drive alone in their cars to work. What does the city conclude about the short term effectiveness of their car pool program? Choose one answer below and show your work.

(i) There is not enough evidence, at the 5% significance level, to conclude that the car pool program is effective in reducing the percentage of workers that drive alone in their cars to work.

(ii) There is evidence, at the 5% significance level, to suggest that the car pool program is effective in reducing the percentage of workers that drive alone in their cars to work.