MATH 218 **FINAL EXAM** **Fall 1999**

INSTRUCTIONS. Every numerical answer should be simplified to a fraction or a decimal, unless otherwise stated. You must show your work and justify your methods to obtain full credit. Use the continuity correction wherever it is appropriate unless otherwise instructed. If you can't do one part of a problem but need that answer later, guess an answer and use that guess for the later part. The exam is worth a total of 200 points.

Problem 1. (15 points) A certain investor is interested in a group of 30 stocks. Among these stocks are 17 stocks traded at the New York Stock Exchange (NYSE) and 13 stocks traded at the American Stock Exchange (Amex): Among the NYSE stocks, the prices of 12 stocks are up since the previous trading day and the prices of 5 stocks are down. Among the Amex stocks, the prices of 7 stocks are up since the previous trading day and the prices of 6 stocks are down. When appropriate, you may leave your answer in terms of factorials.

(a) Suppose one stock is selected at random. Let A represent the event that a stock traded on the NYSE is chosen, and let B represent the event that a stock whose price has risen is chosen. Find $P(A)$ and $P(A|B)$.
(b) Are A and B independent? Show your work.
(c) The investor needs to read a report on each of the Amex stocks whose price is up since the previous trading day, and must choose the order in which to read them. How many ways are there for the investor to do this? Show your work.
(d) How many ways can the investor choose a group of 5 different stocks to buy from the group of 13 Amex stocks?
(e) Suppose that the investor wants to invest in some of the 13 Amex stocks. How many ways can the investor choose a group of 3 Amex stocks that are up and a group of 2 Amex stocks that are down?

Problem 2. (15 points) USC students are either in the college of Letters, Arts and Sciences (LAS), the graduate school (GS) or the professional schools (PS). The percentage of students in each group that support Al Gore for president in the 2000 election are as follows: LAS, 30%; GS, 40%; PS, 20%. There are currently 12000 students enrolled in LAS, 6000 students enrolled in the graduate school, and 9000 students enrolled in the professional schools.

(a) What is the probability that a student chosen at random from the USC student body is enrolled in one of the professional schools?
(b) What is the probability that a student chosen at random from the USC student body will support Al Gore in the 2000 presidential election?
(c) If you pass a student on campus and he or she is wearing a *Gore in 2000* button, what is the probability that the student is enrolled in LAS?

Problem 3. (15 points) The manager of a certain small restaurant has observed that the most popular entree is the fish entree. 30% of the diners select the fish entree.

(a) Find the expected number of fish entrees ordered by a party of 4 diners.
(b) Find the standard deviation in the number of fish entrees ordered by a party of 4 diners.
(c) Find the probability that in a party of 4 diners, exactly 2 of them order the fish entree.

Problem 4. (20 points) An office furniture store is having a sale on a certain type of office chair. The chairs are priced at \$80 per chair. The probability distribution for the number of chairs sold to an individual customer is

Number of chairs	1	2	3	4
Probability	0.15	0.40	0.10	0.35

(a) Find the probability that the number of chairs sold to an individual customer is odd.
(b) Find the expected number of chairs sold to an individual customer.
(c) Find the standard deviation in the number of chairs sold to an individual customer.
(d) The delivery charge for an order of chairs is \$50. Find the expected amount a customer pays for the chairs he or she buys, *including* the delivery charge.
(e) Find the standard deviation in the amount a customer pays for the chairs he or she buys, including the delivery charge.

Problem 5. (15 points) Let X be a continuous random variable denoting the time required for an assembly operation. It is known that X has a uniform distribution between 30 and 40 seconds.

(a) Determine the proportion of assemblies that require more than 37 seconds.
(b) What assembly time is exceeded by 90% of the assemblies?
(c) Determine the mean and standard deviation of the assembly time.

Problem 6. (20 points) The speeds (in mph) of cars on Main Street are observed to be normally distributed with mean 37 and standard deviation 4.5. The speed limit is 35 mph.

(a) What fraction of drivers are exceeding the speed limit?
(b) Officer Krupke decides to give tickets only to the fastest 10% of cars on Main Street. How fast must a driver go to get a ticket from Officer Krupke?

(c) Under the assumptions in (b), find the approximate probability that out of the next 150 cars on Main Street, at most 10 receive tickets from Officer Krupke.

Problem 7. (15 points) A doctor working in a clinic finds that the amount of time he spends with a patient has a distribution with mean 5 minutes and standard deviation 2 minutes. One day he finds that there are 30 patients in the waiting room. Assuming that he sees the patients consecutively with no gaps between them starting at 10 a.m., what is the probability that the doctor will be free for lunch at 12:15 p.m.?

Problem 8. (25 points) The Santa Berenica city council contracted a statistical consultant to prepare a study about the impact of abolishing rent control regulations. As a preliminary step, the researcher decided to estimate the mean rent increase for a one-bedroom apartment in Santa Berenica. From a random sample of 5 one-bedroom apartments in the area he obtained the following monthly rent increases (in dollars): 130, 220, 175, 140, 100.

(a) Find point estimates for the mean and standard deviation of the rent increase for a one-bedroom apartment in Santa Berenica.

(b) Determine a 95% confidence interval for the mean rent increase for one-bedroom apartments in Santa Berenica. (If you need additional assumptions about the rent increases, use them and list them in the next question.)

(c) What assumptions, if any, about the apartment rent increases did you use to answer the previous question?

(d) After the council meeting the consultant was asked to prepare a more detailed study estimating the mean apartment rent increase within $\pm\$10$ with confidence of 99%. How many apartments does he need to select in total (including the original 5) to obtain a confidence interval with the required width?

Problem 9. (20 points) Susie, an 8-year-old budding statistician, observes that whenever her buttered toast falls on the floor, it usually seems to land butter-side-down. (It is buttered on one side.) One day, while her Mom is out, Susie drops her buttered toast on the floor 40 times, and she observes that it lands butter-side-down 27 times.

(a) Find a point estimate for the proportion of time that the toast lands butter-side-down. State whether the estimator you are using is biased or unbiased.

(b) Find Susie's 90% confidence interval for the proportion of time that the toast lands butter-side-down.

(c) Susie's 10-year-old brother Mike is unimpressed—he tells Susie he will determine the probability of butter-side-down to within ± 0.02, with 90% confidence, and will use Susie's data as a mere preliminary sample. How many more times (in addition to Susie's 40) must Mike drop the toast, to achieve his desired accuracy?

Problem 10. (20 points) A researcher for Red Cars of America wants to use a random sample of 426 tickets to see if there is strong evidence to support the idea that an unusually high percentage of tickets go to drivers of red cars. Approximately 11% of the cars on California's roads are red in color.

(a) Formulate appropriate null and alternative hypotheses for the researcher to use.

(b) Choose an appropriate test statistic and rejection rule to test your null hypothesis against your alternative hypothesis at the 5% level of significance.

(c) During a recent 24 hour period, on a particular stretch of freeway, the Highway Patrol issued 426 tickets, of which 63 went to drivers of red cars. Determine whether or not the null hypothesis should be rejected at the 5% level of significance. Explain your answer.

(d) Find the P-value for the data in this test. Your answer may either be an exact value, or else the smallest interval which can be determined from the table provided.

(e) Circle **T**rue or **F**alse: as a result of this hypothesis test, the researcher can conclude:

There is not strong evidence that an unusually high percentage of tickets go to red cars.	T F
Red cars apparently receive an unusually high percentage of tickets.	T F
Drivers of red cars are treated unfairly by the police.	T F

Problem 11. (20 points) FAA regulations state that the cables manufactured for use in aircraft must have a mean length of exactly 40 inches. The FAA investigates a random sample of 15 aircraft cables made by AirTrain, Inc. to test whether the cables meet regulations. If they are either too long or too short, on average, then AirTrain may be fined. Assume that the cable lengths are normally distributed.

(a) Formulate the appropriate null and alternative hypothesis that the FAA investigators should use.

(b) Choose a suitable test statistic and rejection rule to test your null hypothesis against your alternative hypothesis at the 1% level of significance.

(c) In the sample of 15 cables, the average cable length was found to be 39.7 inches, with a sample standard deviation of 0.4 inches. Determine whether the null hypothesis should be rejected at the 1% level of significance. Explain your reasoning.

(d) Find the P-value for the data in this test. Your answer may either be an exact value, or else the smallest interval which can be determined from the table provided.

(e) A second investigation of cables made by another company resulted in a P-value of 0.028. At which of the following significance levels should the null hypothesis be rejected for this second investigation? Circle all that apply.

0.5% 1% 2.5% 5% 10%

Answers to Fall 1999 Math 218 Final Exam

1.: (a) $P(A) = 17/30 = 0.5667$, $P(A|B) = 12/19 = 0.6316$; (b) No, since $P(A) \neq P(A|B)$; (c) 5040; (d) 1287; (e) 525.

2.: (a) 1/3; (b) 13/45 = 0.2889; (c) 6/13 = 0.4015.

3.: (a) 1.2; (b) 0.9165; (c) 0.2646.

4.: (a) 0.25; (b) 2.65; (c) 1.1079; (d) \$262; (e) \$88.63.

5.: (a) 3/10; (b) 31 seconds; (c) 35 seconds, 2.887 seconds.

6.: (a) 0.6700; (b) 40.76 mph; (c) 0.1112 (using the continuity correction).

7.: 0.0853 approximately.

8.: (a) \$153 for mean, \$46.04 for standard deviation; (b) from \$98.84 to \$210.16; (c) assume a normal distribution for monthly rent increases; (d) 164.

9.: (a) 27/40 = 0.675, unbiased; (b) from 0.553 to 0.797; (c) 1445.

10.: (a) $H_0 : p = 0.11$, $H_a : p > 0.11$ where p is proportion of tickets going to drivers of red cars; (b) $Z = (\hat{p} - 0.11)/\sqrt{0.11 \times 0.89/n}$, reject if $Z > 1.645$; (c) data gives $Z = 2.499$ so reject H_0; (d) 0.0062; (e) F, T, F (since possibly drivers of red cars are more dangerous drivers).

11.: (a) $H_0 : \mu = 40$, $H_a : \mu \neq 40$ where μ is the mean length in inches of the cables; (b) $t = (\overline{X} - 40)/(s/\sqrt{15})$, reject if $|t| > 2.977$; (c) data gives $t = -2.904$ so do not reject H_0; (d) between 0.01 and 0.02; (e) do not reject at levels 0.5%, 1% and 2.5%, reject at levels 5% and 10%.