**Math 218** **Final Exam** **Fall 1997**

INSTRUCTIONS. Every numerical answer should be simplified to a fraction or decimal. You must show your work and justify your methods to obtain full credit. Use the continuity correction wherever it is appropriate unless otherwise instructed. If you can't do one part of a problem but need that answer later, guess an answer and use that guess for the later part. The exam is worth a total of 200 points.

*Problem* 1. (20 points) The manager of a brokerage firm with 500 customers asked them to rate their brokers. The results have been tabulated below. The columns describe the customers' incomes and the rows describe their ratings of the brokers.

| | Under \$20,000 | \$20,000 to \$50,000 | Over \$50,000 |
|---|---|---|---|
| Excellent | 50 | 60 | 40 |
| Average | 100 | 120 | 50 |
| Poor | 30 | 35 | 15 |

(a) Find the probability that a randomly selected customer has an income between \$20,000 and \$50,000.
(b) Given that the customer has an income below \$20,000, find the probability that the customer gave the broker an average rating.
(c) Given that the customer has an income of \$20,000 or above, find the probability that the customer gave the broker an average rating.
(d) Determine whether the events 'excellent' and 'over \$50,000' are independent.

*Problem* 2. (25 points) An oil company is about to drill at a particular site. The rock layers at the site may or may not have a 'dome structure'. A dome structure increases the chance of finding oil.

The probability that there is a dome structure at the site is 0.6. If there is a dome structure the probability that the site is dry is 0.6, the probability that the site is a low producer is 0.3 and the probability that the site is a high producer is 0.1. If there is no dome structure the probability that the site is dry is 0.8, the probability that the site is a low producer is 0.15 and the probability that the site is a high producer is 0.05.

If the site is dry the payoff will be $-$\$100,000 (loss of \$100,000); if it is a low producer the payoff will be \$200,000; if it is a high producer the payoff will be \$500,000.

(a) Draw the probability tree.
(b) Find the probability that the site is a low producer.
(c) Given that the site is a high producer, what is the probability that there is a dome structure?
(d) Calculate the expected payoff.

*Problem* 3. (20 points) Today is the $227^{th}$ anniversary of the birth of Beethoven. In honor of that consider the 9 letters in **Ludwig van**.

(a) How many different arrangements of these letters are there? (Ignore the blank space.)
(b) If you were to choose a letter at random from the 9 above, what is the probability that it would be a vowel?
(c) If you were to choose four letters at random (from the 9 above) *with replacement*, what is the probability that two or more of them would be vowels?
(d) If you were to choose four letters at random (from the 9 above) *without replacement*, what is the probability that two or more of them would be vowels?

*Problem* 4. (15 points) It has been observed that the amount of time (in minutes) that a customer at the new Jamba Juice Bar in Cafe 84 has to wait for their fruit shake is a continuous random variable which is uniformly distributed on the interval from 2 to 12.

(a) What is the average time that a customer will have to wait?
(b) Find C such that the probability that a customer will have to wait at least C minutes is 1/4.
(c) Find the probability that a customer will have to wait more than 10 minutes.
(d) If you have lunch at Jamba Juice each day for a week (Monday – Friday), what is the probability that you will have to wait more than 10 minutes at least once during the week?

*Problem* 5. (10 points) A basketball team has an 60% free-throw shooting percentage. Using an appropriate approximation, find the probability that, out of their next 80 free throw attempts, they will make between 50 and 56 (inclusive) of them. Assume the throws are independent.

*Problem* 6. (10 points) Eskimo Electric Corp. manufactures refrigerators. The lifetimes of these refrigerators are normally distributed, averaging 8.5 years with a standard deviation of 1.7 years.

(a) The company guarantees the refrigerators to last at least 6 years. What fraction of all buyers will see their refrigerators fail before the guarantee ends?
(b) How long (at least) do the best 20% of refrigerators last?

*Problem* 7. (15 points) Suppose that the total cost for an employee of Trojan Software to make a business trip to New York averages \$3,200, with a standard deviation of \$1,300. Suppose that 40 such trips are made.

(a) Assume that \$140,000 is budgeted for such trips. What is the probability of running out of money? State whether your answer is exact or approximate.
(b) How much should be budgeted (instead of \$140,000) to reduce this probability to 2%.

*Problem* 8. (25 points) Let $X$ be the travelling time from home to work for a randomly chosen office worker in Los Angeles. Assume that $X$ has a normal distribution. A sample of 6 observations from this population gave the following times (in minutes):

$$35 \quad 20 \quad 25 \quad 15 \quad 22 \quad 21$$

(a) Find point estimates for the population mean $\mu$ and population variance $\sigma^2$.
(b) Find a point estimate for the standard deviation of the sample mean $\overline{X}$.
(c) Find a 95% confidence interval for $\mu$.
(d) Suppose that you are told that $\sigma = 8$. Using this additional information find a 95% confidence interval for $\mu$.

*Problem* 9. (20 points) The manufacturers of *Rise'n'Shine*, a new breakfast cereal, have just finished a major national advertising campaign for the new product, and they wish to determine how effective the campaign has been. In a random sample of 50 supermarket shoppers, 34 of them had seen an advertisement featuring *Rise'n'Shine*.

(a) Find a 95% confidence interval for the proportion $p$ of supermarket shoppers in the whole country who have seen an advertisement for *Rise'n'Shine*.
(b) Based on the preliminary survey, determine how large a sample size is needed in order to estimate $p$ to within $\pm 0.05$ with a confidence level of 95%.

*Problem* 10. (20 points) One of the largest firms making and marketing popular music has changed its manager responsible for acquiring new musical talent. In the past, at least 35% of recording contracts have resulted in hits. The higher level executives wish to find out if this proportion has decreased with the new manager. They decide that they will collect data on how many of the recording contracts made by the new manager have resulted in hits.

(a) State the appropriate null and alternative hypotheses.
(b) Choose a test statistic and rejection rule to test the null hypothesis at the 5% significance level, using a sample of 70 new contracts.
(c) The sample of 70 contracts resulted in 17 hits. Should the null hypothesis be rejected? Clearly explain your reasoning.
(d) Determine the P-value of the result in (c).

*Problem* 11. (20 points) A financial analyst with a major brokerage house specializes in a group of technology stocks. The average price-to-earnings (P/E) ratio of such stocks has been 35. We will assume that the distribution of P/E ratios is approximately normal. The analyst is interested to see if the P/E ratio has changed after the recent activity in the stock market.

(a) State the appropriate null and alternative hypotheses.
(b) Choose a test statistic and rejection rule to test the null hypothesis at the 1% significance level, using a sample of 20 technology stock P/E ratios.
(c) Suppose the average P/E ratio of the twenty stocks chosen is $\overline{X} = 43$, with $s = 15$. Should the null hypothesis be rejected? Clearly explain your reasoning.
(d) The P-value of the result in (c) lies in which of the following intervals? Circle one, and justify your answer.

| | | |
|---|---|---|
| (i) More than .10 | (ii) .05 to .10 | (iii) .02 to .05 |
| (iv) .01 to .02 | (v) .005 to .01 | (vi) less than .005 |

## Answers to Fall 1997 Math 218 Final Exam

**1.:** (a) 0.43; (b) 0.5556; (c) 0.5313; (d) No.
**2.:** (b) 0.24; (c) 0.75; (d) \$20,000.
**3.:** (a) 362,800; (b) 1/3; (c) 0.4074; (d) 0.4048.
**4.:** (a) 7 minutes; (b) 9.5; (c) 0.2; (d) 0.6723.
**5.:** 0.3407 (using the continuity correction).
**6.:** (a) 0.0708; (b) 9.928 years.
**7.:** (a) 0.0721 (approx); (b) \$144,855.
**8.:** (a) 23, 45.2; (b) 2.7447; (c) 15.94 to 30.06; (d) 16.60 to 29.40.
**9.:** (a) 0.551 to 0.809; (b) 335.
**10.:** (a) $H_0 : p \geq 0.35$ and $H_a : p < 0.35$, where $p$ is the proportion of recording contracts made by the new manager which resulted in hits;
(b) $Z = \dfrac{\hat{p} - 0.35}{\sqrt{0.35 \times 0.65/70}}$, reject $H_0$ if $Z < -1.645$;
(c) reject $H_0$ since $Z = -1.879$ is in the rejection region;
(d) 0.0301.
**11.:** (a) $H_0 : \mu = 35$ and $H_a : \mu \neq 35$, where $\mu$ is the average P/E ratio;
(b) $t = \dfrac{\overline{X} - 35}{s/\sqrt{20}}$, reject $H_0$ if $|t| > 2.861$;
(c) do not reject $H_0$ since $t = 2.385$ is not in the rejection region;
(d) .02 to .05.