Name: ____________________

Signature: ____________________

Math 118: Final Exam

Fall 2016

Instructions: This exam has 14 problems, and will last 120 minutes. You may use any standard scientific (non-graphing) calculator. You may also use one 8.5×11 sheet of notes, handwritten, front and back. Read each question completely, show all of your work, and write your solution neatly. Work quickly, but carefully. Before you start, **circle your section below**.

Blois 10am	Haskell 11am	Wolcott 12pm
Blois 12pm	Haskell 1pm	Wolcott 1pm
Tokorcheck 9am	Zhang 10am	Levitt 2pm

Please do not write inside of the following box.

Question 01:	/05	Question 08:	/05
Question 02:	/05	Question 09:	/05
Question 03:	/05	Question 10:	/05
Question 04:	/05	Question 11:	/15
Question 05:	/10	Question 12:	/15
Question 06:	/10	Question 13:	/15
Question 07:	/10	Question 14:	/10
Sum 1:		Sum 2:	

Total Points: ______ /120

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Question 1 (5 points)**.** When someone wins the jackpot, the lottery pays out the winnings at the rate of $5000t$ dollars per year for 20 years (where t is the number of years that have passed since the jackpot was won) making a total of \$1,000,000 paid out. Assuming interest rates are positive, is the present value of the winnings

A. less than \$1,000,000,

B. equal to \$1,000,000,

C. greater than \$1,000,000, or

D. cannot be determined from the information given.

Question 2 (5 points)**.** Suppose that f is decreasing on the interval $[0, 2]$. Using a left-hand Riemann sum, you find that

$$\int_0^2 f(x)\,dx \approx 13.92$$

Which of the following could be the exact value of $\int_0^2 f(x)\,dx$? Circle all that apply.

A. 9

B. 14

C. 28

D. None of the above

Question 3 (5 points)**.** A company sells shiny gizmos for \$125. After conducting extensive market research they determine that the Elasticity of Demand, E, at this price is 0.8. To **increase** the company's **revenue** from selling gizmos the company should:

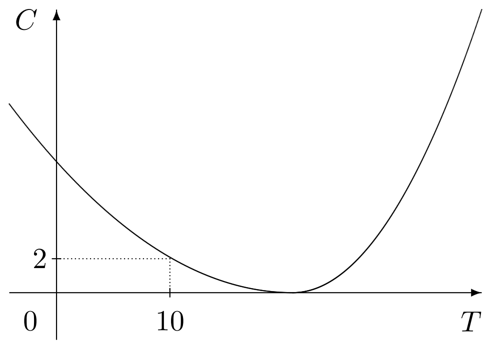
A. increase the selling price,

B. do nothing,

C. decrease the selling price, or

D. it cannot be determined from the information given.

Question 4 (5 points)**.** The daily cost C to heat or cool a house, measured in dollars, is a function of the outside temperature T, measured in degrees celsius. The graph of C as a function of T is shown below.



Suppose when $T = 10$, that $\dfrac{dC}{dT} = -0.2$.

What does this mean in practical terms? Circle only the best answer.

A. When it's 10° outside, the temperature decreases by approximately 0.2° every day.

B. When it's 10° outside, the daily cost to heat the house decreases by approximately \$0.20 every day.

C. When it's 10° outside and the temperature increases by 1°, the daily cost to heat the house decreases by approximately \$0.20.

D. When it's 10° outside and the daily cost to heat the house increases by \$1, the temperature decreases by approximately 0.2°.

This problem is continued as Question 5 on the next page.

Question 5 (10 points)**.** Use the data and graph from the previous problem to answer the following:

(a) Estimate the daily cost of heating the house when it's $10.5°$ outside.

Answer (with units)

(b) Is your estimate an underestimate or an overestimate? Circle your answer and explain.

Underestimate/Overestimate

(c) It's $10°$ outside and as winter approaches, the temperature is decreasing at the rate of $0.7°$ per day. How fast is the daily cost to heat the house rising? Include units in your answer.

Answer (with units)

Question 6 (10 points)**.** Consider the following function $f(x)$:

$$f(x) = -x^3 + 3x^2 + 9x + 2$$

(a) Suppose that f is defined on $(-\infty, \infty)$. Find all **critical points**, and classify each as a local maximum, local minimum, or neither.

Answer

(b) Now suppose that f is defined on the closed interval $I = [-2, 1]$. Find the **global** maximum and minimum of f on I.

Global maximum is ________ and is attained at x = ________.

Global minimum is ________ and is attained at x = ________.

Question 7 (10 points)**.** The Men's Warehouse determines that in order to sell q suits per week, the price per suit (in dollars) must be

$$p = 150 - 0.5q.$$

They also know that the total cost (in dollars) of producing q suits is

$$C = 4000 + 0.25q^2.$$

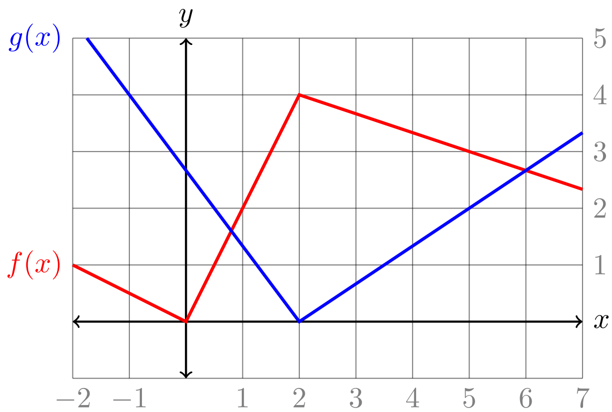
What is the maximum possible profit? Remember to justify that your answer is a maximum.

Maximum profit is ________.

Question 8 (5 points)**.** Over the last century prices have increased, on average, by 3.28% every year. Suppose you want to buy a bottle of Coke that costs \$1.20 today but all you have is a dime (10 cents) and a ride in a time machine that costs a nickel (5 cents). To which year should you travel in your time machine in order to be able to buy the bottle of Coke with the remaining nickel? (Note: Coca-cola has been around since 1886.)

Travel to the year ________.

Question 9 (5 points)**.** The graphs of functions f and g are shown below.



Let $u(x) = f(g(x))$. Find $u'(-1)$.

Answer

Question 10 (5 points). A new video game takes off like wild-fire and sells at a faster and faster rate for the first 12 weeks of its release. The table below shows the rate at which it sells (in thousands of games per week).

weeks since release	0	4	8	12
rate	5.0	8.2	10.5	11.8

Find both upper and lower estimates for the total number of games sold during these 12 weeks.

Upper estimate is ________ thousand games.

Lower estimate is ________ thousand games.

Question 11 (15 points)**.** The marginal cost (in dollars per unit) of producing a product is:

$$MC(q) = \frac{q}{50} + \frac{800}{q+1}.$$

Suppose that the fixed costs are \$8520.

(a) Approximately how much does it cost to produce the 100th unit?

Answer

(b) Suppose that 50 units have already been produced. How much will it cost to produce the next 50 units?

Answer

(c) What is the total cost of producing 100 units?

Answer

(d) On average, how much does it cost **per unit** to produce the first 100 units?

Answer

Question 12 (15 points)**.** Evaluate the following antiderivatives/integrals:

(a) $\displaystyle\int (2x^2 + e^{2x})\, dx$

(b) $\displaystyle\int_0^4 x\sqrt{2x+1}\, dx$

(c) $\displaystyle\int x^2 e^{2x}\, dx$

Question 13 (15 points)**.** Consider the following function:

$$f(x, y) = (y^2 - x^2)e^y.$$

(a) Find all critical points of $f(x, y)$.

(b) Compute all second partial derivatives of $f(x, y)$.

(c) Determine if each critical point is a local maximum, local minimum, or saddle point. If you cannot determine this, explain why.

Question 14 (10 points)**.** Evaluate the following integral:

$$\int_1^3 \int_{-1}^2 \left(x^2 y + \frac{1}{x}\right)\, dy\, dx$$

Answer