MATHEMATICS 126   **FINAL EXAM**   Dec. 18, 2013

Last Name (Print)_____ First Name (Print)_____

USC ID Number _____Signature _____

Circle your Professor's Name

Lin (MWF 9AM)   **Mancera** (MWF 9AM)   **Tiruviluamala** (MWF 11AM, 12 NOON)

Yildirim (MWF 11AM, 1PM)

Directions: **You must show all your work and justify your methods to obtain full credit**.

You can leave expressions in terms of radicals, powers, exponentials and logarithms such as $\ln 5$, e^5, $\sqrt{3}$. Do not use scratch paper. Use the back side of the previous page if additional room is needed. Write your answers in the appropriate places.

No calculators are allowed (or needed). However, you can use a two-sided formula sheet. All cell phones and pagers must be in your backpack, put in the "silent" mode.

Remember USC considers cheating to be a very serious issue.

Do not write on this page below this line.

1. (20)	
2. (20)	
3. (20)	
4. (20)	
5. (20)	
6. (20)	
7. (20)	
8. (20)	
9. (20)	
10. (20)	
Total (200)	

1. Determine whether or not the limit exists. If the limit exists find it, and indicate clearly how you obtained your answer. If the limit does not exist give reasons why.

a) (10 points) $\displaystyle\lim_{x\to 0}\frac{\tan x - x}{x^3}$

$$\lim_{x\to 0}\frac{\tan x - x}{x^3} =$$

b) (10 points) $\displaystyle\lim_{x\to 0^+}\left[\ln\frac{1}{x}\right]^x$

$$\lim_{x\to 0^+}\left[\ln\frac{1}{x}\right]^x =$$

2. Compute the following integrals

a) (7points) $\int x^3 \sqrt{x^2 + 4}\, dx$

$$\int x^3 \sqrt{x^2 + 4}\, dx =$$

b) (7 points) $\int x^2 (\ln x)^2\, dx$

Problem 2 continued.

$$\int x^2 (\ln x)^2 \, dx =$$

c) (6 points) $\displaystyle \int \frac{10}{(x-1)(x^2+9)} \, dx$

$$\int \frac{10}{(x-1)(x^2+9)} \, dx =$$

3. Determine whether the integral is convergent (c) or divergent (d). In case it is convergent find its value.

a) (10 points) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt[3]{x^2}}\, dx$

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Circle your answer:

c   d

</div>

b) (10 points) $\displaystyle\int_{1}^{\infty} \frac{1+e^{-x}}{x}\, dx$

<div style="border:1px solid">

Circle your answer:

c   d

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4. Let $\Re$ be the region of the plane bounded by the curves $y = x^3$ and $y = \sqrt{x}$.

a) (7 points) Sketch the region and find its area.

Area of $\Re$ =

For b) and c) consider the solid of revolution S obtained by rotating $\Re$ about the y-axis.

b) (7 points) Write an explicit integral for the volume V of S using the method of slicing (disks). DO NOT EVALUATE THE INTEGRAL.

c) (6 points) Write an explicit integral for the volume V of S using the method of cylindrical shells. DO NOT EVALUATE THE INTEGRAL.

5. (20 points) A tank has the shape of a half sphere with radius 4 meters. Water is filled in the tank to a height of 3 meters. Set up, but DO NOT EVALUATE, the integral that expresses the work needed to pump all the water to the top of the tank. Use $\rho\left[\frac{kg}{m^3}\right]$ for the water density and $g\left[\frac{m}{s^2}\right]$ for the acceleration of gravity.
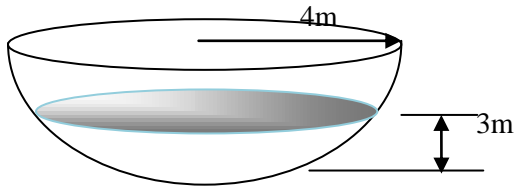
Work =

6. In each case determine whether the series is absolutely convergent (ac) or conditionally convergent (cc) or divergent (d). **(Remember to state clearly any test(s) you use)**

a) (7 points) $\displaystyle\sum_{n=2}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

Circle your answer:
ac   cc   d

b) (6 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^3+4}$

Circle your answer:
ac   cc   d

c) (7 points) $\displaystyle\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{n\sqrt{\ln(n)}}$

Circle your answer:
ac   cc   d

7. a) (8 points) Determine whether the following sequence is convergent (c) or divergent (d). Find the limit, if the sequence is convergent.

$$a_n = (-1)^{n+1} \frac{4n^2 + 1}{5n - n^2}$$

Circle your answer:

c	d

b) (12 points) Find the radius of convergence R, and the interval of convergence for the series:

$$\sum_{n=1}^{\infty} \frac{2^n (x-3)^n}{\sqrt{n+3}}$$

R =   Interval of convergence =

8. a) (13 points) Approximate the function $f(x) = \sqrt[5]{x^2}$ by a Taylor polynomial of degree 2 at $x = 32$.

$$f(x) = \sqrt[5]{x^2} \approx$$

b) (7 points) How accurate is this approximation when $31 \le x \le 33$?

9. a) (8 points) Find the Maclaurin series expansion of $f(x) = x^{10}e^{x^3}$

$$\boxed{f(x) = x^{10}e^{x^3}}$$

b) (5 points) Find $f^{(19)}(0)$.

$$\boxed{f^{(19)}(0) =}$$

c) (7 points) Evaluate $\displaystyle\int_{-1}^{0} x^{10}e^{x^3}\,dx$ as a power series. Write down the first four terms of this series, and determine how many terms are needed in this series to make the error at most 0.01 .

10. Let c be the curve given in polar coordinates by $r = 1 + \cos(\theta)$

a) (10 points) Sketch the curve c

b) (10 points) Find the area of the region inside c.

Area=