**Math 125 • Calculus I** **Name:**________________________________

**Final Exam**, December 17, 2012 **ID:**___________________________________

**Circle your professor and class time if applicable**:

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## INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of $10 \times 20 = 200$ points.

| Question | Points Earned | Points Possible |
|---|---|---|
| Q 1 | | 20 |
| Q 2 | | 20 |
| Q 3 | | 20 |
| Q 4 | | 20 |
| Q 5 | | 20 |
| Q 6 | | 20 |
| Q 7 | | 20 |
| Q 8 | | 20 |
| Q 9 | | 20 |
| Q 10 | | 20 |
| **Totals**: | | 200 |

**1.** a) The following limit represents the derivative of some function $f$ at some number $a$:

$$\lim_{x \to 0} \frac{e^x - 1}{x}.$$

Write down one possible function $f$ and the corresponding value of $a$. What is the limit?

b) Determine whether the following function is continuous at 2.

$$f(x) = \begin{cases} \dfrac{|x-2|}{x^2-4}, & x \neq 2, \\ \\ \dfrac{1}{4}, & x = 2. \end{cases}$$

**2.** Find the limit if it exists. If the limit does not exist, explain why.

(a)

$$\lim_{x\to\infty}\left(\sqrt{x^4+\pi x^2}-x^2\right).$$

(b)

$$\lim_{x\to 0}\left[\sin\left(\frac{2}{x^2}\right)+1\right]|x|.$$

(c)

$$\lim_{x\to 0}\frac{\sin^3 x}{x^2-x}.$$

**3.** Differentiate each of the following:

(a)

$$y = \tan\sqrt{1+x^2},$$

(b)

$$y = (4+x^2)^x,$$

(c)

$$y = e^{x(x^2+1)}.$$

**4.** a) Write down the linear approximation to

$$f(x) = \frac{(1-x)^2}{1+(1+x)^2}$$

at the value $a = 0$.

b) Use the linear approximation from part (a) to estimate $\dfrac{0.99^2}{(1+1.01^2)}$.

**5.** Consider the equation $\sin x - \cos x - 3x = 0$.

(a) Show that the equation has a solution.

(b) Show that the solution is unique.

**6.** Evaluate:

(a)

$$\int_0^1 x(\sqrt{x} - 1)\, dx,$$

(b)

$$\int \frac{\ln(x^2)}{x}\, dx,$$

(c)

$$\int_{-1}^5 |x - 1|\, dx.$$

**7.** a) Differentiate

$$f(x) = \int_{\cos x}^{\sin x} \sqrt{t^4 + 1}\, dt.$$

b) Find $y'$ by implicit differentiation if $y$ satisfies:

$$\ln y = y^2 \ln x.$$

Write down the equation of the tangent line to the curve $\ln y = y^2 \ln x$ at $(1, 1)$.

**8.** In this problem we will graph the function $f(x) = \dfrac{x}{x^2 - 4}$.

(a) State the domain of $f$.

(b) Find any asymptotes of the graph of $f$; if there are none, say so.

(c) We know that $f'(x) = \dfrac{-x^2 - 4}{(x^2 - 4)^2}$ (you do not have to show this). Find the intervals of increase and decrease of $f$.

(d) Locate all local maxima and minima; if there are none, say so.

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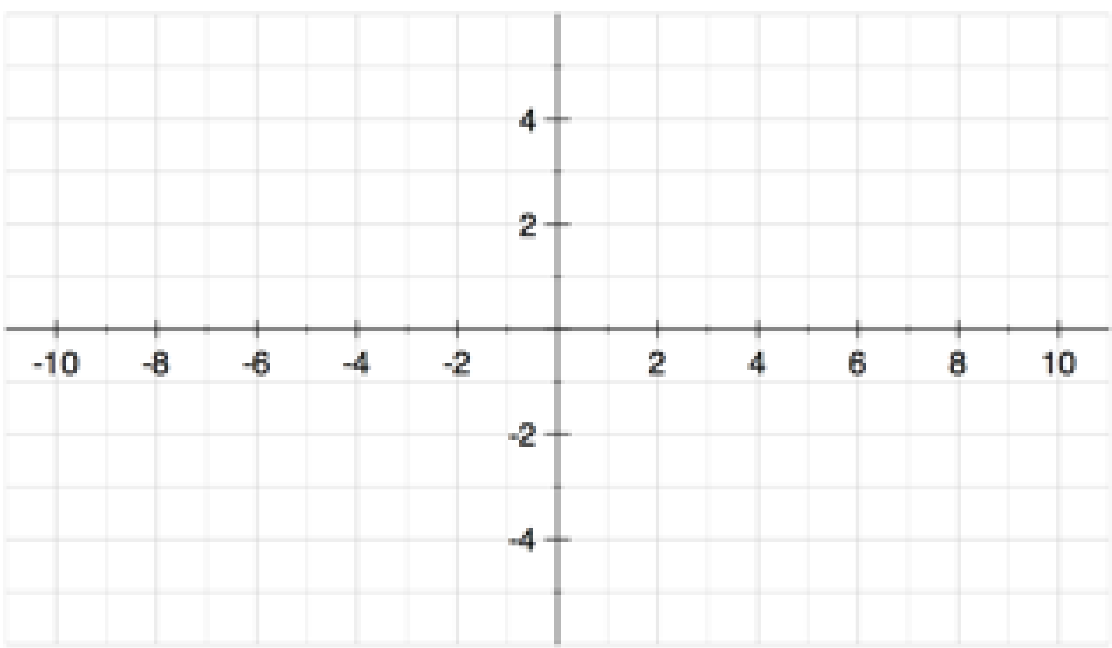
(e) We know that $f''(x) = \dfrac{2x^3 + 24x}{(x^2-4)^3}$ (you do not have to show this). Find the intervals of concavity of $f$.

(f) Locate all inflection points; if there are none, say so.

(g) Sketch the graph on the axes provided.



**9.** A tank is in the shape of an inverted right circular cone with radius 2 m at the top, and depth $6\pi$ m. Suppose water is pumped in at a rate of $1\,\mathrm{m}^3$ per hour. How fast is water level in the tank rising when the water is $2\pi$ m deep?

The volume of a circular cone is given by $V = \frac{1}{3}\pi r^2 h$, where $r$ is the radius of the base and $h$ is the height.

**10.** Suppose the number of bacteria in a culture at time $t$ is given by $N = c(25 + te^{-t/20})$ for some positive constant $c$.

(a) Find the critical values of $N$.

(b) Find the largest and smallest number of bacteria in the culture during the time interval $0 \leq t \leq 100$.