MATH 126 **FINAL EXAM** **December 17, 2012**

INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators and other electronic devices are NOT allowed. Books and lecture notes are NOT allowed. Turn off your cell phones.

You can use one self-prepared handwritten formula sheet (one piece of letter size paper, both sides may be used).

1. [20 points] Determine whether or not the limit exists. If the limit exists find it, and indicate clearly how you obtained your answer. If the limit does not exist give reasons why.

$$\text{(i)}\ \lim_{x\to 3}\frac{x^5-243}{x^3-27}, \qquad \text{(ii)}\ \lim_{x\to\infty}\left(xe^{1/x}-x\right), \qquad \text{(iii)}\ \lim_{x\to 0^+}(\cos x)^{1/x^2}.$$

2. [24 points] Evaluate the integrals.

$$\text{(i)}\ \int x^2\cos(\pi x)\,dx, \qquad \text{(ii)}\ \int\frac{5x-3}{x^3+3x^2}\,dx, \qquad \text{(iii)}\ \int\frac{x^2}{(1+x^2)^{3/2}}\,dx.$$

3. [16 points] Determine whether the integral is convergent or divergent.

$$\text{(i)}\ \int_1^\infty\frac{2+\sin 3x}{\sqrt{1+x+x^3}}\,dx, \qquad \text{(ii)}\ \int_0^2\frac{x}{1-x^2}\,dx.$$

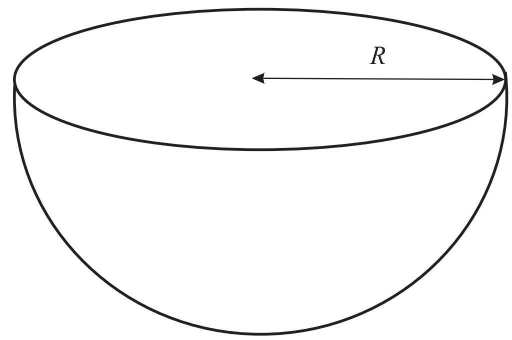
4. [20 points] Consider the region $\mathcal{R}$ between the line $y=3x$ and the curve $y=x^2-2x$.

(i) Sketch the region $\mathcal{R}$ and find its area.

(ii) Set up but **do not evaluate** an integral giving the volume of the solid obtained by rotating the region $\mathcal{R}$ about the y-axis.

(iii) Set up but **do not evaluate** an integral giving the length of the curved part of the boundary of $\mathcal{R}$.

5. [16 points] A hemispherical tank of radius $R = 3$ meters is full of water. Find the work required to pump all of the water out over the rim of the tank. [You may leave your answer in terms of the density of water ρ kg/m^3 and the acceleration due to gravity g m/s^2.]



6. [20 points] The current $I(t)$ in an electric circuit is modeled by the differential equation

$$3\frac{dI}{dt} + 2I = 5.$$

(i) Find the solution $I(t)$ of this differential equation with initial condition $I(0) = 6$.
(ii) Give a sketch of $I(t)$, showing clearly the behavior of $I(t)$ as $t \to \infty$.

7. [20 points] In each case determine whether the series is absolutely convergent or conditionally convergent or divergent. Be sure to state clearly any test(s) you use.

$$\text{(i)} \sum_{n=0}^{\infty} \frac{n \cos n^2}{1+n^3}, \qquad \text{(ii)} \sum_{n=0}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}, \qquad \text{(iii)} \sum_{n=1}^{\infty} \frac{n^2 3^n}{n!}.$$

8. [24 points] [In parts (i), (ii) and (iii) you should give the first three non-zero terms explicitly (with coefficients simplified to simple fractions), as well as an expression for the general term.]

(i) Write down the Maclaurin series for $\dfrac{1}{\sqrt[3]{1+x}}$ and state the radius of convergence.

(ii) Use (i) to obtain the Maclaurin series for $g(x) = \dfrac{x^2}{\sqrt[3]{1+x^2}}$.

(iii) Evaluate the definite integral $\displaystyle\int_0^{1/2} \frac{x^2}{\sqrt[3]{1+x^2}}\,dx$ as an infinite series.

(iv) Determine how many terms of the series in (iii) are needed to approximate the definite integral $\displaystyle\int_0^{1/2} \frac{x^2}{\sqrt[3]{1+x^2}}\,dx$ with an error of at most 1/1000. Be sure to justify your answer.

9. [20 points] (i) Approximate $f(x) = x^{3/4}$ by its Taylor polynomial of degree 2 at $a = 1$.

(ii) Use Taylor's formula to determine the accuracy of this approximation at the point $x = 1/2$. (You do not need to simplify your answer.)

10. [20 points] Let $\mathcal{C}$ be the curve given in polar coordinates by $r = 1 + \cos\theta$, $0 \le \theta \le 2\pi$.
(i) Sketch the curve $\mathcal{C}$.
(ii) Find the area of the region inside $\mathcal{C}$.