

Math 125 Final Exam
Dec 14th, 2011

Directions. Fill out your name, signature and student ID number on the lines below **right now, before starting the exam.** Also, check the box next to the class for which you are registered.

- You must **show all your work and justify your methods** to obtain full credit.
- Your work must be **legible and well-organized** to obtain full credit.
- Write your final answers in the boxes provided.
- Simplify your answers. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$, and $\sqrt{3}$.
- If you continue your work on the back of any page, be sure to **indicate this to the grader.**
- No calculators are allowed, but you may use the sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. It may have anything written on it (on both sides), but it must be written **in your own handwriting.**
- Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes “straying eyes” and failing to stop writing when told to do so at the end of the exam.

Name (please print): _____

Signature: _____

Student ID: _____

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|--|--|---|
| <input type="checkbox"/> 9–10 MWF (Emerson) | <input type="checkbox"/> 11–12 MWF (Haskell) | <input type="checkbox"/> 1–2 MWF (Gupta) |
| <input type="checkbox"/> 10–11 MWF (Rusin) | <input type="checkbox"/> 11–12 MWF (Jedwab) | <input type="checkbox"/> 1–2 MWF (Williams) |
| <input type="checkbox"/> 10–11 MWF (Mancera) | <input type="checkbox"/> 12–1 MWF (Iovanov) | <input type="checkbox"/> 2–3 MWF (Williams) |

Do not write on this page below this line.

1 (20 pts)	6 (25 pts)
2 (20 pts)	7 (30 pts)
3 (20 pts)	8 (15 pts)
4 (20 pts)	9 (15 pts)
5 (20 pts)	10 (15 pts)

200 points total

Problem 1. (20 points) Calculate the following limits. Write $+\infty$ or $-\infty$ if appropriate.

a) i) $\lim_{x \rightarrow 0} \left(\left[\sin x + \cos \left(\frac{1}{x} \right) \right] x^2 \right)$

a) i)

ii) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{|x - 1|}$

a) ii)

iii) $\lim_{x \rightarrow -\infty} \left(x + \sqrt{x^2 + 2x + 3} \right)$

a) iii)

b) Consider the function f that is given by

$$f(x) = \begin{cases} x + 1 & x < 1 \\ c & x = 1 \\ 5x^2 - 4 & x > 1 \end{cases}$$

For what value(s) of c is the function continuous at $x = 1$? Write 'none' if there are none. Explain.

b)

Problem 2. (20 points) Find the derivative $f'(x)$ in each part. You do not need to simplify your answers.

a) $f(x) = e^{\left(\frac{x}{1+x}\right)}$

a) $f'(x) =$

b) $f(x) = (2 + \cos x)^x$

b) $f'(x) =$

c) $f(x) = \ln [(x^2 + a^2)^5(x + b)^6(x + c)^7]$ where a , b and c are constants.

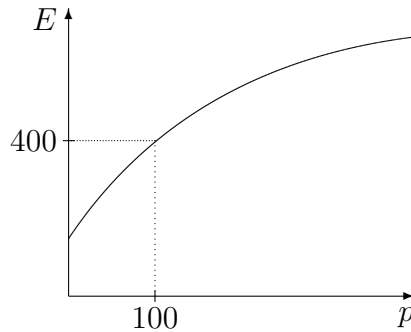
c) $f'(x) =$

d) $f(x) = \int_x^{x^{2011}} e^{-t^2} dt$

d) $f'(x) =$

Problem 3. (20 points) The energy consumption of a small town E (measured in MW) is a function of the population p (measured in thousands of people). The graph of E against p is shown below. Notice that $E = 400$ when $p = 100$. Suppose, furthermore, that

$$\left. \frac{dE}{dp} \right|_{p=100} = 1.2$$



- a) Use calculus to estimate the energy consumption of the town when $p = 110$. Include units in your answer.

a)

- b) Is your estimate in a) an underestimate or an overestimate? Circle your answer and explain.

Underestimate

Overestimate

- c) At present the town has 100 thousand people, but the population is increasing at the rate of 0.5 thousand people per year. How fast is the energy consumption of the town increasing? Include units in your answer.

c)

Problem 4. (20 points) Consider the set of all rectangles that lie in the first quadrant, have one edge along the positive x -axis, one corner at the origin, and the diagonally opposite corner on the curve $y = e^{-x}$. Find the dimensions of the rectangle in the set that has the largest area.

Dimensions are:

Problem 5. (20 points)

a) Evaluate $\int \frac{x^3 - x^2 + x^{3/2}}{\sqrt{x}} dx$

a)

b) Evaluate $\int_0^{\sqrt{\pi}} x \sin(x^2 + \pi) dx$

b)

c) Evaluate $\int \frac{e^x - e^{-x}}{(e^x + e^{-x})^2} dx$

c)

Problem 6. (25 points) Consider the equation

$$x^4 + 4x - 3 = 0.$$

a) Show that the equation has at least two real solutions.

b) Show that the equation has at most two real solutions.

Problem 7. (30 points) Consider the function f that is given by

$$f(x) = \frac{6}{x^2 + 3}.$$

Of all lines tangent to the graph of f , find the line of maximum slope. Be sure to carefully justify that the line you found is the one whose slope is a global (and not just a local) maximum.

Value of x where tangent has a maximum slope:

Equation of tangent line with maximum slope:

Problem 8. (15 points) Consider the curve that is given by the equation

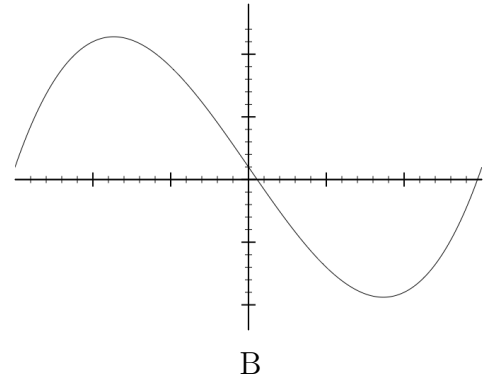
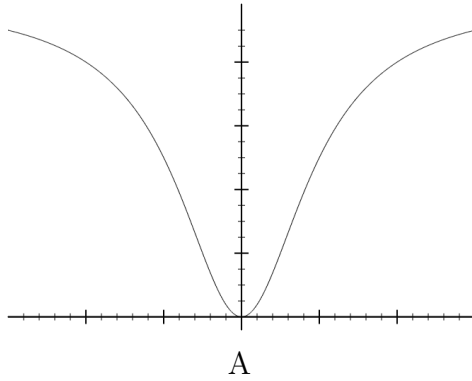
$$2(x^2 + y^2)^2 = 25xy .$$

Use implicit differentiation to find the equation of the tangent line at the point $(1, 2)$.

Equation of tangent line:

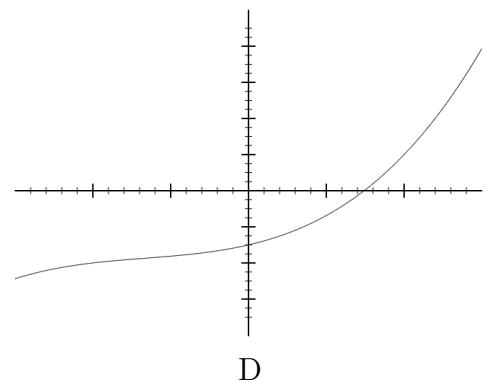
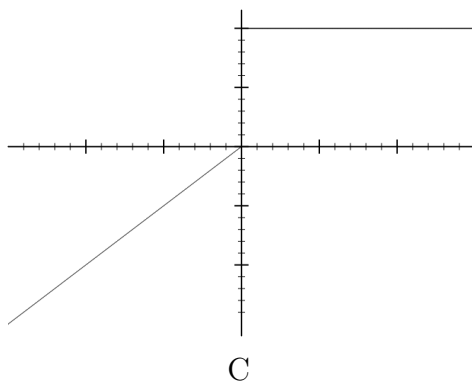
Problem 9. (15 points) In each part below, the graphs of two functions are shown. One of them has the indicated property and the other does not. Which function has the property? Circle your answer in the box and explain. The functions are all graphed on the interval $[-3, 3]$.

a) A function that has more critical points than inflection points on the interval $[-3, 3]$.



<p>Circle your answer: A or B</p> <p>Explain:</p>
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b) A function f whose fourth derivative $f^{(4)}$ has the property that $f^{(4)}(x) = 0$ for all $x \in (-3, 3)$.

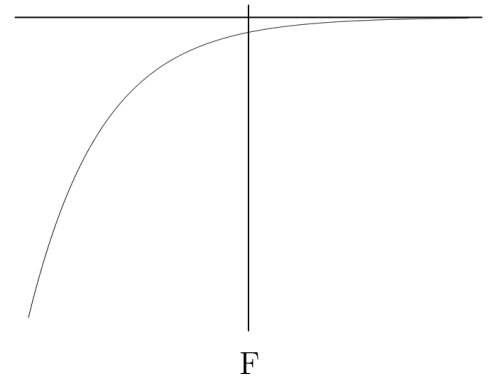
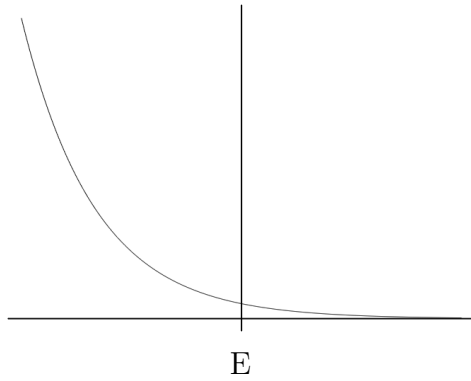


<p>Circle your answer: C or D</p> <p>Explain:</p>
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This problem is continued on the next page.

Problem 9 continued.

c) A function f satisfying $f'(x) = |f(x)|$ for all $x \in (-3, 3)$.



Circle your answer: E or F

Explain:

Problem 10. (15 points) Suppose that the acceleration of a particle (in m/sec^2) at time t (in sec) is given by the function

$$a(t) = 6t - 6$$

for $t \geq 0$.

- a) Find the velocity $v(t)$ of the particle as a function of time if $v(0) = -9$ m/sec.

$v(t) =$

- b) Find the net displacement of the particle from $t = 0$ to $t = 4$. Include units in your answer.

Displacement =

- c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$. Include units in your answer.

Distance traveled =