MATH 126 **FINAL EXAM** **December 14, 2011**

INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators and other electronic devices are NOT allowed. Books and lecture notes are NOT allowed. Turn off your cell phones.

You can use one self-prepared handwritten formula sheet (one piece of letter size paper, both sides may be used).

1. (i) Find real numbers a and r such that

$$\lim_{x\to\infty} ax^r \left(\frac{\pi}{2} - \arctan x\right) = 1.$$

(ii) Determine whether or not the limit exists. If the limit exists find it, and indicate clearly how you obtained your answer. If the limit does not exist give reasons why.

$$\lim_{x\to 0^+} (1+\sin 3x)^{2/x}$$

2. Evaluate the integrals.

(i) $\displaystyle\int \sqrt{x}\,\ln x\,dx$

(ii) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

(iii) $\displaystyle\int \frac{2x^2+x-6}{x^3+3x^2}\,dx$

3. In each case determine whether the integral is convergent or divergent. Remember to justify your answers.

(i) $\displaystyle\int_1^\infty \frac{x}{e^x+x^2}\,dx$

(ii) $\displaystyle\int_2^5 \frac{1}{(5-x)^{2/3}}\,dx$

4. (i) Sketch the region $\mathcal{R}$ bounded by the curve $y = x(1-x)$ and the x axis.

(ii) Set up, but do not evaluate, an integral giving the volume of the solid obtained by rotating the region $\mathcal{R}$ about the x-axis.

(iii) Set up, but do not evaluate, an integral giving the volume of the solid obtained by rotating the region $\mathcal{R}$ about the line $x = 2$.

5. Find the solution of the differential equation

$$(1+\cos x)\, y' = (1+e^{-y}) \sin x$$

with initial condition $y(0) = 0$.

6. In each case determine whether the series is convergent or divergent. Remember to justify your answers.

(i) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 \sin n}{1+n^4}$

(ii) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(iii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{2n}}{(1+3n^2)^n}$

7. Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^n}{2^n(1+\sqrt{n})}.$$

8. (i) Write down the Maclaurin series for $f(x) = x^2 \sin x$. You should give the first four non-zero terms in the series explicitly, as well as an expression for the general term.

(ii) Find $f^{(17)}(0)$.

(iii) Using the Maclaurin series from (i), evaluate the definite integral $\displaystyle\int_0^1 f(x)\, dx$ with an error of at most 0.001. You may leave your answer as a sum of fractions.

9. (i) Find the Taylor polynomial of degree 2 for the function $f(x) = 1/(3x+1)$ at $a = 1$.

(ii) Use Taylor's formula to determine the accuracy of this approximation at the point $x = 0.9$. (You do not need to multiply out your answer.)

10. Let $\mathcal{C}$ be the curve given in polar coordinates by $r = \theta$, $0 \leq \theta \leq \pi$.

(i) Sketch the curve $\mathcal{C}$.

(ii) Find the area of the region bounded by $\mathcal{C}$ and the x-axis.