Math 125 Final Exam
Dec 15th, 2010

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam! Also, check the box next to the class for which you are registered.

- You must **show all your work and justify your methods** to obtain full credit.
- Write your final answers in the boxes provided.
- Simplify your answers. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$, and $\sqrt{3}$.
- If you continue your work on the back of any page, be sure to indicate this to the grader!
- No calculators are allowed, but you may use the sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. You may have anything written on it (on both sides), but it must be written *in your own handwriting.*
- Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Name (please print): ______________________

Signature: ______________________

Student ID: ______________________

☐ 9–10 MWF (Haskell)	☐ 11–12 MWF (Iovanov)	☐ 1–2 MWF (Emerson)
☐ 10–11 MWF (Goldstein)	☐ 11–12 MWF (Rusin)	☐ 1–2 MWF (Montgomery)
☐ 10–11 MWF (Haskell)	☐ 12–1 MWF (Gupta)	☐ 2–3 MWF (Rusin)

Do not write on this page below this line!

1 (12 pts)	7 (12 pts)
2 (18 pts)	8 (26 pts)
3 (24 pts)	9 (20 pts)
4 (14 pts)	10 (20 pts)
5 (12 pts)	11 (12 pts)
6 (10 pts)	12 (20 pts)

200 points total

Problem 1. Calculate the following limits. Write $+\infty$ or $-\infty$ if appropriate.

a) $\lim_{x \to 5} \dfrac{x^2 - 6x + 5}{x^2 - 5x}$

b) $\lim_{x \to \infty} \left(\sqrt{x^2 + x + 1} - x\right)$

Problem 2. Find the derivative of each function.

a) $f(x) = \tan^2(x^3 + 1)$

b) $f(x) = x^x$

c) $f(x) = \displaystyle\int_1^{x^2} e^t \sin t \, dt$

Problem 3. Evaluate the following integrals:

a) $\displaystyle\int_{-\pi/4}^{\pi/4} \left(\cos\theta + \sec^2\theta\right) \, d\theta$

b) $\displaystyle\int_0^1 \frac{x+1}{x^2 + 2x + 3} \, dx$

c) $\displaystyle\int x^5 \sqrt{1 + x^3} \, dx$

Problem 4. Consider the curve that is given by the equation

$$e^x + \ln y = x^2 - y^2 + 2 \ .$$

Find the equation of the tangent line at the point $(0, 1)$.

Problem 5. The population of the city of Metropolis grows exponentially. In 2000, the population was 2 million. In 2010, the population was 3 million. Let P denote the population (measured in millions of people) and let t denote the number of years that have passed since the year 2000.

a) Find an expression for P as a function of t.

b) In what year did the population reach 2.5 million?

Problem 6. Consider the function f that is given by

$$f(x) = \begin{cases} 3x & x < 1 \\ x^2 + x & x \geq 1 \end{cases}$$

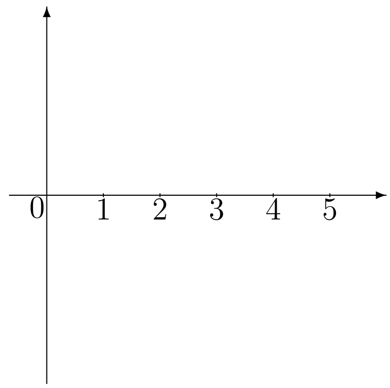
a) Is f continuous at $x = 1$? Answer yes or no and explain.

b) Is f differentiable at $x = 1$? Answer yes or no and, if it is, find $f'(1)$, and if it is not, explain why not.

Problem 7. On the axes below, sketch the graph of a function that has all of the following properties.

- It's defined everywhere on the interval $[1, 5]$.
- It has local minima at $x = 2$ and $x = 4$ and no other local minima.

- It has no local or global maxima.

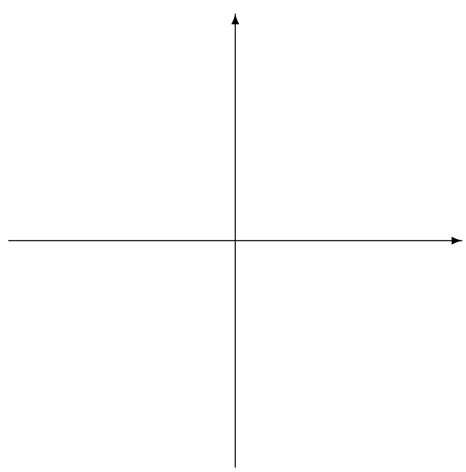


Problem 8. Consider the function f given below. Its first and second derivatives are also provided (you do not need to derive these yourself).

$$f(x) = \left(\frac{x}{1+x}\right)^2 \qquad f'(x) = \frac{2x}{(1+x)^3} \qquad f''(x) = \frac{2-4x}{(1+x)^4}$$

Find all of the following information about f. Don't leave any information blank; instead, write 'none' if appropriate.

a) Domain.
b) Vertical and horizontal asymptotes.
c) All the intervals where f is increasing and all the intervals where f is decreasing.
d) Both the x and y coordinates of any local maxima and minima.
e) All the intervals where f is concave up and all the intervals where f is concave down.
f) Both the x and y coordinates of any points of inflection.
g) Sketch the graph of f on the axes below. Your graph should clearly reflect all the information you found above.



Problem 9.

a) Show that for any $x \geq 0$,

$$e^x \geq 1 + x \ .$$

(*Hint:* you may want to apply the mean value theorem to the function $f(x) = e^x$.)

In parts b) and c) consider the function g that is given by

$$g(x) = e^x - \frac{x^2}{2} - 2, \text{ where } x \geq 0.$$

You may find it helpful to recall that $e \approx 2.7$.

b) Show that g has at least one root.
c) Show that g has at most one root. (*Hint:* use part a).)

Problem 10. The perimeter of a rectangular copper sheet is 12 inches. The two opposite edges are welded together to form a cylindrical pipe. What dimensions of the rectangle will result in a cylinder of maximum volume?

Problem 11. Find

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \frac{1}{1 + (i/n)} \ .$$

(*Hint:* express the limit as a definite integral.)

Problem 12. Santa Claus needs to budget his time in order to be able to deliver presents to all the world's children. In order to deliver presents to Billy's city he needs

$$T = \frac{3p^{3/2}}{p + 4}$$

seconds, where p is the population of the city measured in hundreds of thousands of people. The city presently has 400,000 people in it (i.e. $p = 4$).

a) How many seconds does it now take for Santa to deliver presents to Billy's city?
b) Billy's city is presently growing at the rate of 8,000 (i.e. 0.08 hundred thousand) people per year. How fast is the amount of time that Santa needs to deliver presents to Billy's city increasing? Remember to include units in your answer. (*Hint:* use related rates.)
c) Santa can't afford to spend more than 4.2 seconds delivering presents to Billy's city. Use differentials to estimate in how many years his delivery time will reach his time limit of 4.2 seconds.