Math 125 - Final Exam — December 16, 2009

Last Name________________________ **First Name**________________________

Student ID Number_______________ **Signature**___________________________

Circle your instructor's name

Bonahon   Chen (11am)   Chen (2pm)

Goldstein   Haskell   Kamienny   Tuffaha

Instructions

Answer all questions. You must show your work to obtain full credit. Calculators are not allowed. If you need more space, write on the back of another page and clearly mark on your problem that it is continued elsewhere.

Problem	Points	Score
1 (a) & (b)	14	
1 (c) & (d)	14	
2	15	
3	20	
4 (a) & (b)	12	
4 (c) & (d)	12	
5	21	
6	24	
7	24	
8	20	
9	24	
Total	200	

1. Compute the following integrals

a) $\displaystyle\int_1^2 \frac{x^2+x+1}{x^2}\,dx$

b) $\displaystyle\int_0^\pi \tan\left(\frac{x}{3}\right)\,dx$

c) $\displaystyle\int \frac{3^{\tan x}}{\cos^2 x}\, dx$

d) $\displaystyle\int x^5\sqrt{x^3+4}\, dx$

2. Write the limit

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(1 + \frac{i}{n}\right)^5$$

as an integral, and compute the integral.

3. Let $f(x) = \dfrac{e^x}{x^e}, \quad x > 0.$

a) Find the critical numbers of f, and determine the intervals where f is increasing, and the intervals where f is decreasing.

b) Which is larger, e^π or π^e? Explain.

4. Evaluate the limit. You may not use L'Hopital's rule.

a) $\lim\limits_{x \to -3^+} \dfrac{x^2 - 9}{x + 3}$

b) $\lim\limits_{t \to -\infty} \dfrac{\sin(-t)}{t}$

c) $\lim_{x \to 0} \quad \dfrac{\tan x}{x}$

d) $\lim_{x \to \frac{\pi}{4}} \quad \dfrac{\sin x - \sqrt{2}/2}{x - \dfrac{\pi}{4}}$

(Hint: Interpret the limit as a derivative.)

5. Find the derivative of each function

a) $f(x) = \dfrac{\tan x}{x^2 + 1}$

b) $f(x) = x \cdot e^{\sin x}$

c) $f(x) = \int_{2x}^{x^2} \ln(\cos t)\, dt.$

6. A rectangular box is designed to have a square base and an open top. The material for the bottom costs 2 dollars per square foot, while the material for the sides costs 3 dollars per square foot.

   a) If the box must have a volume of 9 ft^3 find the dimensions of the box that would minimize the cost of the materials.

   b) If the cost of the box is to be 216 dollars find the dimensions that maximize the volume of the box.

7. Sketch the graph of the function f that satisfies

a) f is continuous for all $x \neq 0, 2$.

b) f has a vertical asymptote at $x = 2$.

c) $\lim_{x \to 0^+} f(x) = 4$

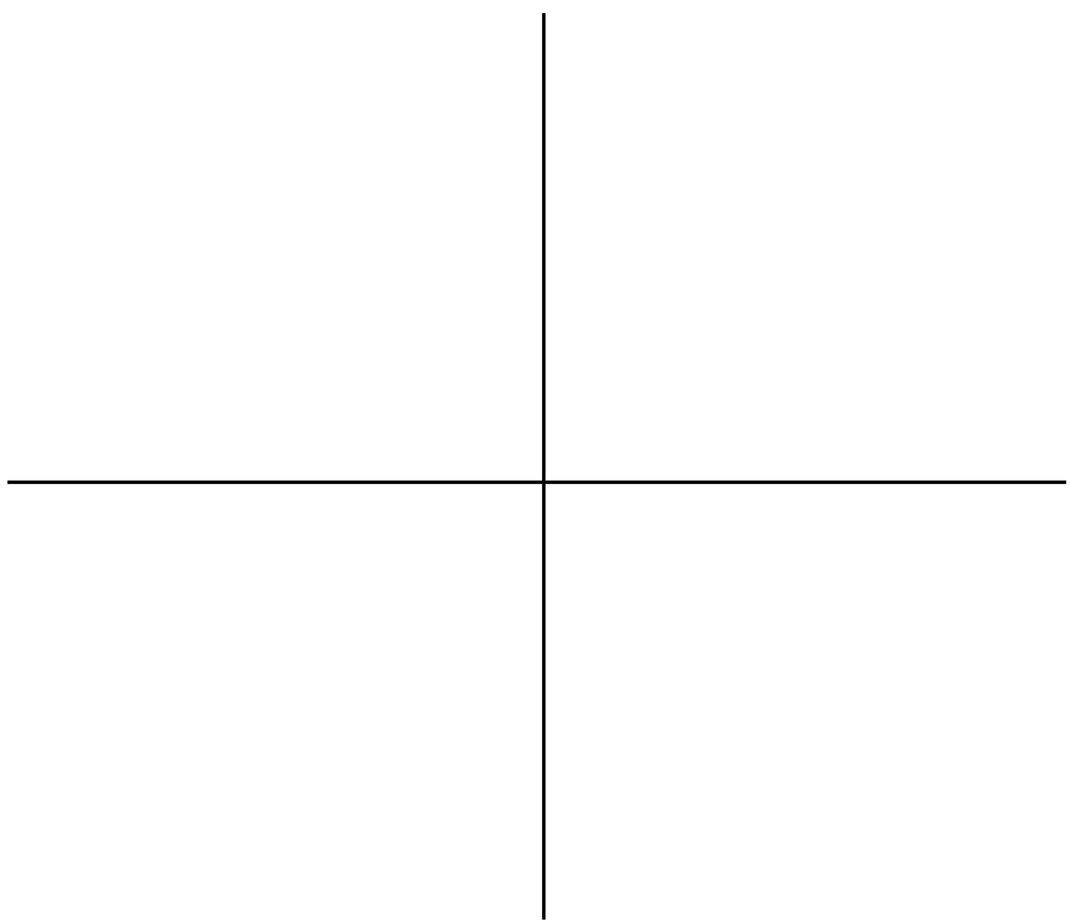
d) $\lim_{x \to 0^-} f(x) = 3$

e) $\lim_{x \to \infty} f(x) = 1$

f) $\lim_{x \to -\infty} f(x) = 1$

g) $f'(x) > 0$ on $(-\infty, 0)$ and $(2, \infty)$

h) $f'(x) < 0$ on $(0, 2)$



8. The value of a car V (in thousands of dollars) depends on how far it has been driven m (in thousands of miles) and its age t (in years). For a certain car it is estimated that

$$V = 100\frac{1 + 15e^{-m/100}}{99 + t^2}$$

   a) Alex's parents bought the car for him new a year ago but he only just recently got his license, so the car is one year old but doesn't yet have any miles on it. If Alex starts driving it now at the rate of 10 thousand miles a year, how fast will its value decrease?

   b) Use your answer to a) to estimate how much the car will be worth two years from now.

9. Consider the curve that is given by $x^3 + xy^3 = 9(y - 1)$. Notice that the point (1, 2) lies on this curve.

   a) Find an equation of the tangent line to the curve at the point (1, 2).

   b) Estimate the y-coordinate of that point on the curve that is close to (1, 2) and whose x-coordinate is equal to 0.9.