MATH 126 - Fall '09

Final Exam

Name: **Student Number:**

Please read all of the following rules carefully before proceeding.

- Check that this Exam contains 11 pages.
- Unless otherwise instructed, please clearly indicate all work involved in the solution of each problem. You will receive partial credit for partial progress toward a solution.
- You may use one 8 x 11 in letter paper with notes (both sides); you may not refer to any other books or notes during the course of the exam.
- You may **not** use a calculator on the exam.

Problem	Possible	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

Please encircle the name of your instructor:

A.Asok **D.Kim,** **M.Iovanov** **T.Sakai** **A.Tuffaha**

Problem 1 (10 pts.)

Evaluate each of the following integrals

1. $$\int x \sin^2(2x)\, dx$$

2. $$\int \frac{2x+1}{x^2-7x+12} dx$$

Problem 2 (10 pts.)

Evaluate each of the following limits. Use the symbols $+\infty$ and $-\infty$ whenever appropriate. If a limit does not exist, state why.

1. $$\lim_{x \to 0} \frac{\tan^2 x + 2x}{x^2 + x}$$

2. $$\lim_{x \to \infty} (xe^{1/x} - x)$$

3. $$\lim_{x \to 0} \frac{e^x + 1}{x^2}$$

Problem 3 (10 pts.)

A region enclosed by $y = \sin x$ and $y = 0$ for $0 \leq x \leq \pi$ is rotated about $x = -\pi/2$. Find the volume of the resulting solid. Sketch the region being rotated, with the coordinate axes and the axis of rotation to support your answer.

Problem 4 (10 pts.)

All the materials required to build a pyramid are located at ground-level. The pyramid is built by laying a foundation and lifting material from the ground onto the part already erected. The base of the pyramid on the ground is a square with side 15 m, and the height of the pyramid is 10 m. Find, but DO NOT EVALUATE, an integral that expresses the total work required to lift the material to build the pyramid. Let ρ (measured in kg/m^3) be the density of the material. Draw a sketch showing the pyramid, the origin and the coordinate axis to support your answer. (**Note**: you can use $g = 10\ \text{m/s}^2$ for the gravitational constant, or you can leave your answer in terms of ρ and g).

Problem 5 (10 pts.)

Determine the value of the integral

$$\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}},$$

if it exists.

Problem 6 (10 pts.)

Find the sum of the series or show that the series is divergent.

1. $\sum_{n=0}^{\infty} \frac{5^n - 2}{7^n}$.

2. $\sum_{n=3}^{\infty} \frac{1}{n(n-1)}$.

Problem 7 (10 pts.)

Determine whether the series converges or diverges. Justify your answer (state clearly any tests you use).

1. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n+9}}$.

2. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n^2+5}{2n^2+1}\right)^n$.

Problem 8 (10 pts.)

If $T_n(x)$ is the n-th Taylor polynomial for $\sin(x)$ around $x = 0$, what n is sufficient for $T_n(1)$ to approximate $\sin(1)$ with an error less than 0.001?

Problem 9 (10 pts.)

Solve the initial value problem

$$\frac{dy}{dx} = xye^{x^2}; y(0) = 1.$$

Problem 10 (10 pts.)

Find the area of the region enclosed by one loop of the curve $r = \sin(4\theta)$.