MATH 126 **FINAL EXAM** **December 17, 2008**

Last Name: _____ **First Name:** _____

Student ID Number: _____ **Signature:** _____

Circle your instructor's name:

Bene (11AM) Bene (1PM) Bonahon Crowley Mancera

INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. You may use a "cheat sheet". The exam is worth a total of 200 points.

Problem	Points	Score
1	20	
2ab	20	
2cd	20	
2e	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	20	
9	20	
10	20	
11	10	
12	10	
Total	200	

Problem 1. Compute the following limits, if they exist. (They may be finite or infinite).

a) (10 points) $\lim_{x\to 0} \dfrac{\cosh 2x - 1}{\sinh(x^2)}$

b) (10 points) $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}}$

Problem 2. Compute the following integrals

a) (10 points) $\displaystyle\int \frac{1}{1+\mathrm{e}^{x}}\,dx$

b) (10 points) $\displaystyle\int x\tan^{-1}x\,dx$

Problem 2 continued.

c) (10 points) $\displaystyle\int \frac{\sin^4 x}{\cos^6 x}\, dx$

d) (10 points) $\displaystyle\int \frac{1}{(x^2+4)^{\frac{5}{2}}}\, dx$

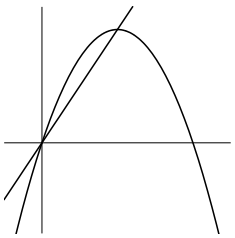
Problem 2 continued.

e) (10 points) $\displaystyle\int \frac{4}{x^2(x+2)}\,dx$

Problem 3. (10 points) You want to approximate the integral $\int_1^2 \mathrm{e}^{x^2} dx$ by using the Trapezoidal Rule with n intervals. How large should you take n to guarantee that the error is at most 10^{-6}? (Do not compute the approximation.)

Problem 4. (10 points) Determine whether the integral $\int_2^3 \frac{1}{\sqrt{3-x}} dx$ is convergent or divergent. Evaluate it if it is convergent.

Problem 5. (10 points) Compute the volume of the solid obtained by revolving about the y–axis the region in the xy-plane enclosed by the curves $y = \frac{1}{2}x$ and $y = x - x^2$.



Problem 6. (10 points) A well is 100 ft deep, and a water bucket is hauled from the bottom to the top, using a rope which weighs 0.1 pounds per foot. The water bucket weighs 40 pounds. How much work is needed to bring the bucket and the rope to the top of the well?

Problem 7. (10 points) Find a function $y = y(x)$ such that $\dfrac{dy}{dx} = 3x^2\sqrt{1-y^2}$ and $y(0) = \frac{1}{2}$.

Problem 8. Determine whether each of the following sequences converges or diverges.

a) (10 points) $a_n = \dfrac{(\ln n)^2}{n}$

b) (10 points) $b_n = (-1)^n(1 - \dfrac{1}{n})$

Problem 9. Indicate whether each of the following series converges or diverges. Determine the sum if it converges.

a) (10 points) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

b) (10 points) $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n}{3n+1}$

Problem 10. (20 points) Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}(x+3)^n$.

Problem 11. (10 points) Write the first 6 terms of the Taylor approximation of $\cos x$ at $a = \frac{\pi}{4}$.

Problem 12. (10 points) Find the area of the region of the plane consisting of those points which are inside of the cardioid of polar equation $r = 1 + \cos\theta$ and to the right of the vertical line of equation $x = \frac{3}{4}$.