MATH 125 **FINAL EXAM** **December 19, 2007**

Last Name: ______________________ **First Name:** ______________________

USC ID: ______________________ **Signature:** ______________________

Circle the lecture section you are registered for:

Voineagu at 9 Tuffaha at 9 Gundersen at 10 Jaffrey at 11
Montgomery at 11 Proskurowski at 12 Geisser at 12 Jaffrey at 1

INSTRUCTIONS

Read the problems carefully and answer the questions asked. Write neatly and indicate clearly your answer to each problem. The backs of the sheets may be used for scratch paper or to continue your work on a problem, but if you do continue your answer, please give directions to the grader.

You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Calculators, notes, books, or collaboration with others are *not* allowed. If you have any questions about any of the problems ask the proctor, but no one else!

Problem	Value	Score
1	21	
2	28	
3	21	
4	15	
5	20	
6	20	
7	15	
8	20	
9	15	
10	25	
Total	200	

1. Find the limits, if they exist (use any method, except L'Hospital's rule):

   (a) $\lim_{x \to 16} \frac{4 - \sqrt{x}}{16 - x}$

   (b) $\lim_{x \to 1} \left(\frac{1}{x - 1} + \frac{1}{x^2 - 3x + 2} \right)$

   (c) $\lim_{t \to 0} \frac{t^3}{\tan^3(t)}$

2. Find the derivative of the following functions:

(a) $f(x) = \dfrac{\sin(2x)}{3x^3 - x}$

(b) $f(x) = xe^{\sqrt{3x}}$

(c) $f(x) = (\ln(5x))^3$

(d) $f(x) = \int_0^{\ln x} \sqrt{t^5 + 1}\, dt$

3. Evaluate the following integrals. Simplify your answer.

(a) $\int_1^8 \frac{(1-\sqrt[3]{x})}{\sqrt{x}}\, dx.$

(b) $\int e^{3\cos(x)} \sin(x) dx.$

(c) $\int \frac{x^3}{x^2+1} dx$

4. (a) Give a definition of the definite integral $\int_a^b f(x)\,dx$ as a limit of Riemann sums.

(b) Consider $L = \lim_{n\to\infty} \left(\frac{1}{n}\right)\left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \cdots + \left(\frac{n}{n}\right)^9\right]$.

View the limit L as a limit of Riemann sums on the interval [0,1], then calculate L by evaluating the corresponding integral.

5. Consider the equation $x^4 + 6x^2 - 5 = 0$.

(a) Show that the equation has at least two solutions.

(b) Show that the equation has at most two solutions.

6. Find the dimensions of the rectangle of largest area which has its base on the positive x-axis, one of its sides on the y-axis, and a vertex on the curve $y = e^{-x^2}$. Justify your answer.

(the graph of $y = e^{-x^2}$ was given in the exam)

7. The population of a certain bacteria grows with time t as

$$p(t) = \frac{1}{1 + Ae^{-kt}},$$

where A and k are positive constants.

(a) Find $\lim_{t \to \infty} p(t)$

(b) Find the rate of population growth and evaluate it at $t = 0$.

8. Find the equation of the tangent line to the curve

$$(x^2 + y^2)^{3/2} = 2xy$$

at the point $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

9. (a) Give the linear approximation $L(x)$ of a function $f(x)$ at a point $x = a$.

(b) Find $L(x)$ when $f(x) = \sqrt{x}$.

(c) Estimate $\sqrt{35.9}$ using $a = 36$ in (b).

10. This problem concerns the graph of the function

$$f(x) = (x^2 - \frac{5}{4})e^{-x}.$$

Its first and second derivative are given by

$$f'(x) = (-x^2 + 2x + \frac{5}{4})e^{-x} \text{ and } f''(x) = (x^2 - 4x + \frac{3}{4})e^{-x}.$$

Find the following, if they exist:

(a) asymptotes, vertical ____________________ or horizontal ____________________

(b) intervals where increasing____________________ or decreasing____________________

(c) local maxima or minima____________________

The problem is continued on the next page.

10. (continued)

$$f(x) = (x^2 - \frac{5}{4})e^{-x}, \ \ f'(x) = (-x^2 + 2x + \frac{5}{4})e^{-x} \ \text{ and } \ f''(x) = (x^2 - 4x + \frac{3}{4})e^{-x}.$$

(d) intervals where concave up__________________ or down__________________

(e) points of inflection__________________

(f) Sketch the graph on the axes provided.