MATH 126 **FINAL EXAM** **December 19, 2007**

Read the problems carefully and answer the questions asked. Write neatly and indicate clearly your answer to each problem. The backs of the sheets may be used for scratch paper or to continue your work on a problem. You are to do all ten problems. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Calculators, notes, books, or collaboration with others are *not* allowed. If you have any questions about any of the problems ask the proctor, but no one else!

1. Determine whether or not the limit exists. If the limit exists find it, and indicate clearly how you obtained your answer. If the limit does not exist give reasons why.

$$\text{(i)}\ \lim_{x\to 2}\frac{x^7-128}{x^2-4}, \qquad \text{(ii)}\ \lim_{x\to 0}(\cos x)^{1/x^2}.$$

2. Evaluate the integrals.

$$\text{(i)}\ \int_0^{1/2}\arcsin x\ dx, \qquad \text{(ii)}\ \int\frac{x^3}{\sqrt{1+x^2}}\ dx, \qquad \text{(iii)}\ \int\frac{1+x}{x+x^3}\ dx.$$

3. Determine whether the integral is convergent or divergent

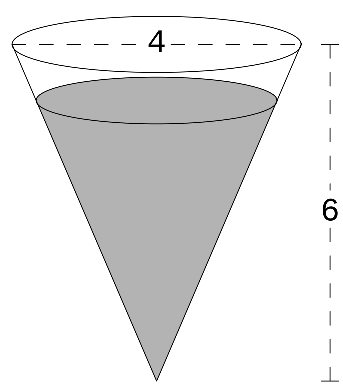
$$\text{(i)}\ \int_1^{\infty}\frac{2+\sin x}{\sqrt[3]{1+x^2+x^4}}\,dx, \qquad \text{(ii)}\ \int_0^{\pi/6}\frac{\cos x}{\sin^2 x}\,dx.$$

4. Consider the region $\mathcal{R}$ bounded by the function $y=\sqrt{x}$, the x-axis and the line $x=1$.

(i) Set up but do not evaluate an integral giving the volume of the solid obtained by rotating the region $\mathcal{R}$ about the line $x=2$.

(ii) Set up but do not evaluate an integral giving the volume of the solid obtained by rotating the region $\mathcal{R}$ about the line $y=1$.

5. A tank is in the shape of a circular cone with its point downwards. The height of the tank is 6 meters and the diameter of the top rim of the tank is 4 meters. The tank contains water to a depth of 5 meters. Calculate the work required to pump just enough water out over the rim of the tank so as to reduce the depth of water to 3 meters. [The density of water is 1000 kg/m^3.]



6. Determine whether the series is convergent or divergent. Remember to justify your answers.

$$\text{(i)} \sum_{n=1}^{\infty} \frac{n^2+2}{(n+1)^4}, \qquad \text{(ii)} \sum_{n=2}^{\infty} \frac{1}{n \ln n}, \qquad \text{(iii)} \sum_{n=1}^{\infty} (-1)^n \cos(1/n).$$

7. Find the sum of the series or show that the series is divergent.

$$\text{(i)} \sum_{n=0}^{\infty} \left(\frac{\pi}{2}\right)^{n+1} \left(\frac{3}{5}\right)^{2n}, \qquad \text{(ii)} \sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!}.$$

8. Find the radius of convergence *and* the interval of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(-5)^n (x-2)^n}{\sqrt{n+1}}.$$

Find also the value $f^{(3)}(2)$.

9. (i) Approximate the function $f(x) = x^{1/2}$ by its Taylor polynomial of degree 2 at $a = 16$.

(ii) Use Taylor's formula to determine the accuracy of this approximation at the point $x = 15$. (You do not need to simplify your answer.)

10. Sketch the curve given in polar coordinates by

$$r = 1 + \cos\theta$$

and find the area of the region enclosed by the curve.