MATH 125 **FINAL EXAM** **December 13, 2006**

Last Name: ______________________ **First Name:**______________________

Student ID Number: ____________________ **Signature:**____________________

Circle your instructor's name:

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INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of 200 points.

Problem	Points	Score
1	18	
2	18	
3	24	
4	18	
5	18	
6	18	
7	18	
8	16	
9	18	
10	16	
11	18	
Total	200	

Problem 1. Compute the following limits, if they exist. (They may be finite or infinite). You may **not** use l'Hôpital's rule, if you know what that is. You must justify your answers.

a) (6 points) $\lim_{x \to -1} \dfrac{x^2 - 3x - 4}{x + 1}$

b) (6 points) $\lim_{x \to 0} \dfrac{\sin \sqrt[3]{x}}{x}$

c) (6 points) $\lim_{x \to -\infty} \dfrac{\sin^2 x}{x\sqrt{x^2 + 1}}$

Problem 2. Find the derivative of the following functions.

a) (6 points) $f(x) = \dfrac{1+\ln x}{1-\ln x}$

b) (6 points) $f(x) = \tan^3(2x+5)$

c) (6 points) $f(x) = \displaystyle\int_1^{\ln x} \mathrm{e}^{u^2}\,du$

Problem 3. (24 points) Consider the function

$$f(x) = x^{\frac{2}{3}}(x-5).$$

The first and second derivatives of $f(x)$ have already been computed for you (you do not need to check this):

$$f'(x) = \frac{5(x-2)}{3x^{\frac{1}{3}}} \text{ and } f''(x) = \frac{10(x+1)}{9x^{\frac{4}{3}}}.$$

Answer the following questions, by writing your answers on the dotted lines. (Note that the problem continues on the next page).

- Intervals where the function is increasing: ....

- x–coordinates of critical points of $f(x)$: ....

- x–coordinates of local maxima of $f(x)$: ....

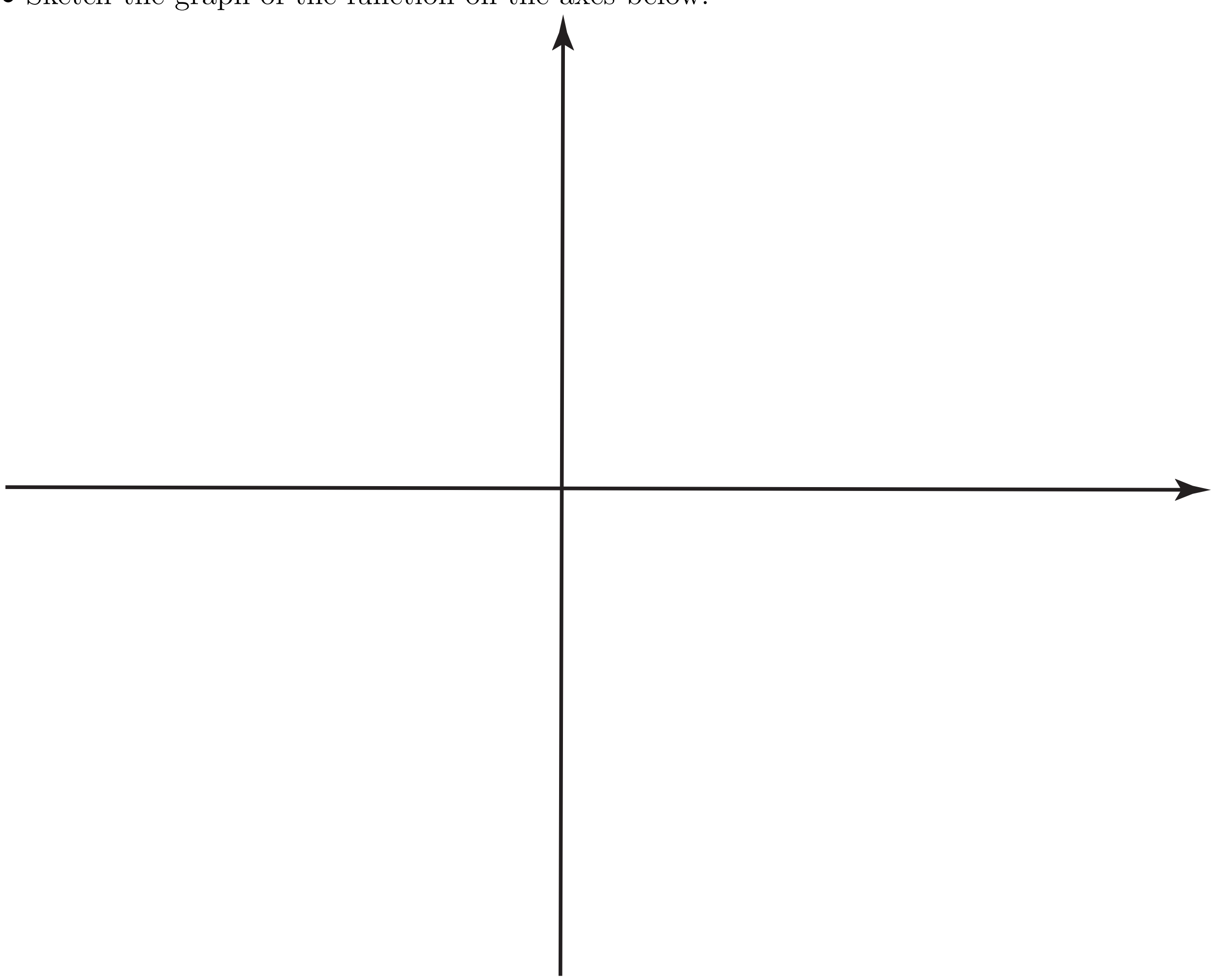
- x–coordinates of local minima of $f(x)$: ....

Problem 3 continued.

- Intervals where the function is concave upward: ....

- x–coordinates of inflection points of $f(x)$: ....

- Sketch the graph of the function on the axes below.



Problem 4. (18 points) Consider the ellipse defined by the equation $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$. Find the points on the ellipse that are closest to the point $(-1, 0)$.

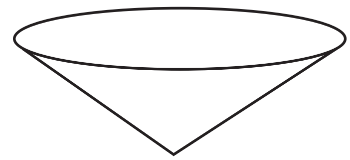
Problem 5. (18 points)

The pressure P and the volume V inside a spherical soap bubble satisfy the equation $PV = K$, where K is a constant. Initially, the radius of the bubble is 6 cm. If the pressure P increases by 0.5% (in other words, if $\Delta P = 0.005P$), estimate the change in the radius of the bubble.

(We remind you that the volume enclosed by a sphere of radius r is $\frac{4}{3}\pi r^3$.)

Problem 6. (18 points) In the xy–plane, consider the curve given by the equation $y = \mathrm{e}^{xy}$. Find the equation of the line tangent to the curve at the point $(-\mathrm{e}, \frac{1}{\mathrm{e}})$.

Problem 7. (18 points) Water is poured into a tank shaped as an inverted cone of radius 15 m and of height 10 m. The water flows at the rate of 2 m^3 per minute when the water is 6 m deep. How fast is the water level rising at that time?



Problem 8.

a) (10 points) Write the following limit as an integral.

$$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \sqrt{4 - \left(\frac{2i}{n}\right)^2}$$

b) (6 points) Compute the limit by interpreting the integral as an area.

Problem 9. Evaluate the following integrals. Make sure that you simplify your answer.

a) (6 points) $\displaystyle\int \frac{\sin t}{\cos t + 12} dt$

b) (6 points) $\displaystyle\int 2x^3\sqrt{x^2+1}\,dx$

c) (6 points) $\displaystyle\int_1^{e^4} \frac{\sqrt{\ln x}}{x} dx$

Problem 10. Consider the equation $x^5 + 3x = 1$.
a) (8 points) Show that the equation has at least one solution between 0 and 1. Make sure that you explain why.

a) (8 points) Show that the equation has only one solution between 0 and 1. Make sure that you explain why.

Problem 11. The body of a murder victim was found at noon in a room that is kept at 20°C. From Newton's law of cooling, it is known that the temperature H of the body varies as $H = A + Be^{-kt}$, where t is the time elapsed since the body was found and where A, B and k are positive constants.
a) (4 points) Find the value of A. (Hint: In the long run, the body will reach room temperature.)

b) (8 points) When the body was found, its temperature was 35°C. One hour later, its temperature was 31°C. Use this information to find B and k.

c) (6 points) Assuming that, at the time of the murder, the victim's body had the normal body temperature of 37°C, when did the murder occur? You do not need to simplify your answer.