Math 126 **Final exam** **December 13, 2006**

Last Name: _____ **First Name:** _____

Student ID Number: _____ **Signature:** _____

Instructions: Use the space provided for each answer. If needed, use the back of the page, always indicating where the solution is continued. Please write legibly and make sure it's clear to the reader what your answer is. To receive full credit, show and explain all your work. Whenever possible, try to simplify your answer. Calculators are not allowed. The exam is worth a total of 200 points.

Please circle your instructor's name:

Bene

Kalman

Lototsky (9 am)

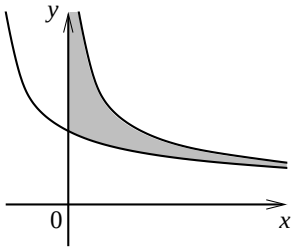
Lototsky (10 am)

Mikulevicius

Problem	Value	Score
1	20	
2	20	
3	15	
4	20	
5	35	
6	10	
7	20	
8	20	
9	20	
10	20	
Total	200	

1. (20 points) A monkey is 10 ft of the ground in a tree with a long rope. She spots a bunch of bananas directly below on the ground and manages to "lasso" the bananas with the rope. If the bunch of bananas weighs 15 lbs, and the rope weighs 2 lbs per foot, how much work is done by the monkey in pulling up the bunch of bananas?

2. (20 points) Consider the region between the two curves $y = 1/\sqrt{x}$ and $y = 1/\sqrt{x+1}$ in the first quadrant.



(a) Use the washer method to set up an integral for the volume of the solid obtained by rotating the region for $x \geq 1$ around the x–axis.

(b) Use the shell method to set up an integral for the volume of the solid obtained by rotating the region for $0 \leq x \leq 1$ around the y–axis.

(c) Evaluate one of the two integrals above.

3. (15 points) Suppose we rotate the curve $y = \sqrt{x}$, $2 \leq x \leq 6$ about the x-axis. Find the area of the resulting surface of revolution.

4. (20 points) Consider the curve given in polar coordinates as $r = e^{-\theta}$, $\theta \geq 0$.

   (a) Noting that $e^{-\theta}$ is a positive and decreasing function of θ, draw a rough sketch of the curve for $\theta \in [0, 4\pi]$.

   (b) What is the length of the curve as it turns from $\theta = 0$ to $\theta = 2\pi$?

   (c) Determine whether the total length of the curve for $\theta \geq 0$ is finite, and if so, find it.

5. Find the following integrals.

(a) (10 points) $\int \frac{x+1}{x^2-4x+6}\,dx$

(b) (15 points) $\int_0^\infty x^{-3}e^{-1/x}\,dx$

(c) (10 points) $\int \frac{t^3}{\sqrt{4-t^2}}\, dt$

6. (10 points) Find

$$\lim_{n\to\infty} \frac{n\cos(n)}{e^n}.$$

Carefully justify your answer.

7. (20 points) Decide whether the following infinite series are absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n(n-1)/2}\cdot\left(\frac{n}{2n-1}\right)^n$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\cdot\frac{\ln n}{n}$

8. (20 points) Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)} (x-2)^n.$$

9. (20 points) Find the Taylor expansion of the function

$$g(x) = \frac{1}{x^2 - 7x + 12}$$

at the point $a = 0$. Make sure you write the general term of the expansion. Then compute $g^{(53)}(0)$. (Suggestion: use partial fractions and geometric series.)

10. (20 points)

(a) Find the Taylor expansion of the function $f(x) = \int_0^x \sin(t^2)\,dt$ at the point $a = 0$. Make sure you write the general term of the expansion.

(b) Find an upper bound on the error $|f(1) - T_9(1)|$, where T_9 is the ninth degree Taylor polynomial of f at zero.