MATH 126 **FINAL EXAM** **December 14, 2005**

You should do all ten problems. Read the problems carefully and answer the questions asked. Write neatly and indicate clearly your answer to each problem. If you need more space, use the backs of these pages and clearly indicate where the continuation may be found. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Calculators, notes, books, or collaboration with others are *not* allowed. If you have any questions about any of the problems ask the proctor, but no one else!

1. [16 points] (a) Sketch the region $\mathcal{R}$ in the first quadrant below the curve $y = 6 + x - x^2$.
(b) Find the volume of the solid $\mathcal{V}$ obtained by rotating the region $\mathcal{R}$ about the y-axis.

2. [24 points] Evaluate the integrals

(i) $\displaystyle\int x \sin x \, dx$

(ii) $\displaystyle\int \frac{3x^2 - 3x + 4}{(x-2)(x^2+1)} \, dx$

(iii) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}} \, dx$

3. [24 points] Determine whether or not the limit exists. If the limit exists find it, and indicate clearly how you obtained your answer. If the limit does not exist give reasons why.

(i) $\displaystyle\lim_{x \to 0} \frac{\sin 3x}{5x + 7x^2}$

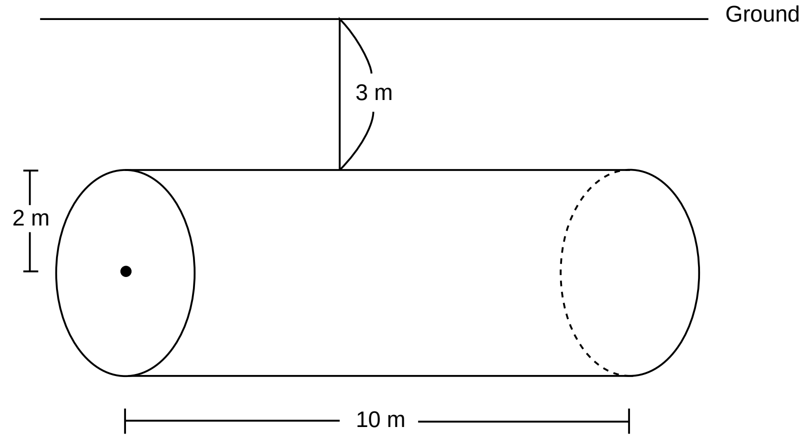
(ii) $\displaystyle\lim_{x \to \infty} x\big(e^{5/x} - 1\big)$

(iii) $\displaystyle\lim_{x \to 0} \frac{\sin x - x}{\sqrt{1 + x^3} - 1}$

4. [16 points] Does $\displaystyle\int_{-\infty}^{\infty} e^{-x^2} \, dx$ converge or diverge? Clearly explain your reasoning.

5. [20 points] Consider the curve given in polar coordinates by $r = \sin^3(\frac{\theta}{3})$ for $0 \le \theta \le 2\pi$.
(a) Sketch the curve.
(b) Find the length of the curve.

6. [20 points] An underground tank is full of gasoline. The tank has the shape of a right circular cylinder, drawn as in the figure. (It is sitting sideways, its radius is 2m and the length is 10m. The top of the tank is 3m below the ground.) Set up, but do not evaluate, an integral which gives the work required to pump all the gasoline up to the ground. Be sure to define all your variables. Use ρ to denote the density of gasoline in kg/m^3 and g to denote the acceleration due to gravity in m/s^2.



7. [24 points] Determine whether the series is convergent or divergent. You should justify your answer and indicate the test you are using.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\ln n}$

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

(iii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

(iv) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

8. [16 points] Find the interval of convergence of the power series $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{n(x-3)^n}{4^n}$.

9. [20 points] (a) Write down the power series expansion for $\sqrt{1+x^3}$ centered at $a = 0$. Make sure you write down the expression for the general term.

(b) Write down the first three nonzero terms in the power series for $f(x) = \displaystyle\int_0^x \sqrt{t^3+1}\, dt$, centered at $a = 0$.

10. [20 points] (a) Approximate the function $f(x) = x^{1/3}$ by its Taylor polynomial of degree 3 at $a = 1$.

(b) Use Taylor's inequality to determine the accuracy of this approximation on the interval $0.5 \le x \le 1.5$.