1. (5 points each) Compute the following limits, if they exist. You may **not** use l'Hôpital's rule, if you know what that is. You must also justify your answer.

   (a) $\displaystyle\lim_{x\to+\infty} \frac{3x^2+1}{(1-x)(2x-1)}$

   (b) $\displaystyle\lim_{x\to 0} x\cot 7x$

   (c) $\displaystyle\lim_{x\to 1} \frac{\sqrt{5-x}-\sqrt{4x}}{x^2-1}$

2. (6 points each) Find $\dfrac{dy}{dx}$ of the following functions. (You do not need to simplify your answer.)

   (a) $y = \sin\left(\ln(1+x^{100})\right)$

   (b) $y = x^e e^x$

   (c) $y = \displaystyle\int_0^{\sqrt{x}} (1+e^{-t^2})\, dt$

   (d) $y = \dfrac{\tan x}{1+\sec x}$

3. (6 points each) Evaluate the following integrals. (You do not need to simplify your answer.)

   (a) $\displaystyle\int \left(\frac{5x^2+1}{x} + \frac{2}{\cos^2 x} - \sin x\right)\, dx$

   (b) $\displaystyle\int_0^1 x\cos(1+x^2)\, dx$

   (c) $\displaystyle\int 5^x\sqrt{1+5^x}\, dx$

4. (12 points) Knowing that $\sqrt[4]{81} = 3$, use a linear approximation to estimate $\sqrt[4]{85}$.

5. (15 points) Find an equation of the tangent line to the curve $xe^{2x} - y^2\ln y = 0$ at the point $(1, e)$.

6. (7 points each) Consider the function

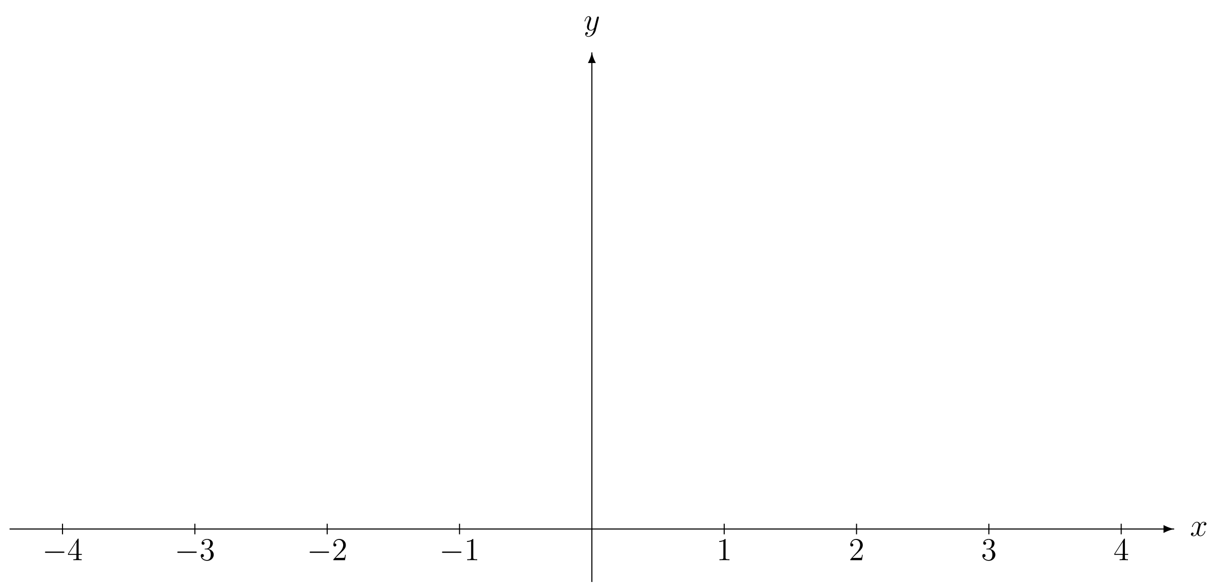
$$f(x) = \begin{cases} x\cos\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   (a) Is f continuous at $x = 0$? Explain your answer.

(b) Is f differentiable at $x = 0$? Explain your answer.

7. (30 points) Sketch the graph of the function $f(x) = 2 - \frac{2x}{(1+x^2)^2}$, using the axes provided, and fill in the required information below. If none, write NONE. The derivatives of $f(x)$ have already been computed for you (you do not need to check this):

$$f'(x) = \frac{6x^2 - 2}{(1+x^2)^3} \quad \text{and} \quad f''(x) = \frac{-24x^3 + 24x}{(1+x^2)^4}.$$



(a) Horizontal asymptotes (if any):. . . . .

(b) Vertical asymptotes (if any): . . . .

(c) x-coordinates of local maxima (if any): . . . .

(d) x-coordinates of local minima (if any): . . . .

(e) x-coordinates of inflection points (if any): . . . .

(f) Intervals where f is concave upward (if any): . . . .

(g) Intervals where f is concave downward (if any): . . . .

8. (12 points) Find an approximation to the integral

$$\int_{-1}^{2} \ln(x^2 + 1)\ dx,$$

using a Riemann sum with left endpoints and $n = 6$ subintervals. You do not need to simplify your answer.

9. (12 points) Find the absolute maximum and absolute minimum values of the function

$$f(x) = 2x^2 - x^4$$

on the closed interval $[-\frac{1}{2}, 2]$.

10. (24 points) Water is being pumped from a square pond, each of whose sides is 20 meters long, to a circular pond whose radius is 10 meters. If the water level in the square pond is dropping at the rate of 0.3 meters per hour, how fast is the water rising in the circular pond?

11. (24 points) Find the largest possible area of a rectangle inscribed in the ellipse

$$x^2 + \frac{y^2}{4} = 1,$$

whose sides are parallel to the x-axis or the y-axis.