MATH 126 **FINAL EXAM** **December 15, 2004**

Read the problems carefully and answer the questions asked. Write neatly and indicate clearly your answer to each problem. The backs of the sheets may be used for scratch paper or to continue your work on a problem. All problems count equally and you are to do all ten problems. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Calculators, notes, books, or collaboration with others are *not* allowed. If you have any questions about any of the problems ask the proctor, but no one else!

1. Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$.

(i) Calculate the area of $\mathcal{R}$.

(ii) Calculate the volume of the solid obtained by rotating $\mathcal{R}$ about the y-axis.

2. A circular swimming pool has a diameter of 36 ft and sides that are 6 ft high, and the depth of the water is 5 ft. Find the work required to pump the water out over the side. (Use the fact that water weighs 62.5 lb per cubic ft. You may leave your answer as a product. Include the units.)

3. Evaluate the integrals

(i) $$\int x^2 \cos x \, dx$$

(ii) $$\int \frac{1}{x^2\sqrt{x^2+1}} \, dx$$

(iii) $$\int \frac{4x+5}{x^2+x-2} \, dx$$

4. Determine whether or not the limit exists. If the limit exists find it, and indicate clearly how you obtained your answer. If the limit does not exist give reasons why.

(i) $$\lim_{x \to 0} (1 + 3x + 5x^2)^{1/x}$$

(ii) $$\lim_{x \to \pi/4} (1 - \tan x) \sec 2x$$

5. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(i) $$\int_{-\infty}^{\infty} \frac{x^2}{9+x^6} \, dx$$

(ii) $\displaystyle\int_0^1 \frac{(\ln x)^2}{x}\,dx$

6. (i) Sketch the curve given in polar coordinates by $r = 1 + \cos\theta$.

(ii) Find the equation of the tangent line to the curve at the point given by $\theta = \pi/2$.

(iii) Find the length of the curve.

7. Determine whether the series is convergent or divergent.

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{3 - \sin n}{n^{3/4} + 2}$

(ii) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 5n^2 + 3}$

(iii) $\displaystyle\sum_{n=1}^{\infty} \tan(1/n^2)$

8. Find the radius of convergence *and* the interval of convergence of the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n\sqrt{n+1}}.$$

9. (i) Use a geometric series as a step in writing down the Maclaurin series for the function

$$f(x) = \frac{x}{1 + x^4}$$

and state its radius of convergence.

(ii) Use the result of (i) to estimate $\displaystyle\int_0^{1/2} f(x)\,dx$ correct to within 10^{-4}. (You may leave your answer as a sum or difference of fractions.)

10. (i) Approximate the function $f(x) = \ln(1 + 2x)$ by its Taylor polynomial of degree 2 at $a = 1$.

(ii) Use Taylor's inequality to estimate the accuracy of this approximation on the interval $0.5 \le x \le 1.5$.