Math 125 - Final(Common) - Fall 2003

1. [20 points] Find the limits, if they exist, of the following expressions (you may not use L'Hospital's rule).

1a. $\lim_{x\to 0} x\cot x$
1b. $\lim_{x\to\infty}(\sqrt{x}-x)$
1c. $\lim_{x\to 0}\dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$.

2. [20 points] Consider

$$f(x)=\begin{cases}\dfrac{x^2-x}{|x-1|} & \text{if } x\neq 1\\ \\ 2 & \text{if } x=1.\end{cases}$$

2a. Sketch the graph of $f(x)$.
2b. Find the numbers at which $f(x)$ is **discontinuous**. If none, write NONE.
You must justify your answer.

3. [15 points] If a snowball melts so that its surface area ($S=4\pi r^2$) decreases at a rate of 1 cm^2/min, find the rate at which the radius decreases when the radius is 10 cm.

4a. [10 points] Find the linear approximation to $f(x)=e^x$ near $x=0$.
4b. [5 points] Sketch the graph of f and its linear approximation in the x interval of $[-1,1]$.

5. Find the derivatives of the following functions:

5a. [5 points] $f(x)=\dfrac{x^3+1}{x^2+5}$
5b. [5 points] $f(x)=3e^{2x^2+1}$
5c. [5 points] $f(x)=\ln(1+x^2)\sin x$

6.[10 points] Use implicit differentiation to find the slope of the tangent line to the curve

$$x^2+xy+y^2=3$$

at the point $(1,1)$.

7. [25 points] Given $f(x)=2(\ln x)^2$, $x>0$, find (state the complete answer)

7a. vertical and horizontal asymptotes (if any);
7b. intervals of increase and decrease;
7c. local minimum and maximum values (if any);
7d. intervals of concavity and inflection points (if any). **7e.** Sketch the graph.

8. [25 points] Suppose you wish to make a rectangular box with a square base from two different materials. The material for the top and four sides costs \$1/sq.ft. while the material for the base costs \$2/sq.ft. Find the dimensions of the box of greatest volume if you are allowed to spend \$144 for all the material.

9. [10 points] A piecewise linear function is given by

$$f(x) = \begin{cases} -3 & \text{if} \quad x \leq -2 \\ 3x/2 & \text{if} \quad -2 \leq x \leq 4 \\ 6 & \text{if} \quad x \geq 4 \end{cases}$$

Evaluate the following definite integrals: [HINT: draw a graph, think of areas of triangles and rectangles.]

9a. $\int_{-10}^{-2} f(x)\ dx$

9b. $\int_{-2}^{2} f(x)\ dx$

9c. $\int_{-10}^{2} f(x)\ dx$

9d. $\int_{0}^{4} f(x)\ dx$

10. [10 points] Express the area, under the curve $y = 3x^2 + 4x - 2$, above the x-axis, and between the lines $x = 1$ and $x = 2$, as a definite integral. Evaluate, and simplify — the area is a whole number.

11a. [5 points] Consider $G(x) = \int_{0}^{x} t^3 \sin t\ dt.$

Find $G'(x) =$

11b. [5 points] Consider $H(x) = \int_{0}^{x^2} t^3 \sin t\ dt.$

Find $H'(x) =$

12. Evaluate the following integrals (give your answers in a simplified form).

12a. [10 points] $\int (2\,x^3 + 5\,\sqrt{x} + 3\,\cos(x) - 10\,x^{-1} + 8\,e^{-2\,x})dx$

12b. [5 points] $\int_{0}^{\pi/4} \cos(2\,x)\,(\sin(2\,x))^3\,dx$

12c. [5 points] $\int \frac{3x}{(x^2+2)} dx$

12d. [5 points] $\int \frac{3x}{(x^2+2)^5} dx$