MATH 126 **FINAL EXAM** **12/17/03**

Read the problems carefully and answer the questions asked. Write neatly and indicate clearly your answer to each problem. All problems count equally and you are to do all ten problems. You must show your work to obtain full credit. Calculators, note, books, or collaboration with others are *not* allowed.

1. Let T be the bounded region between the graphs of $y = x$ and $y = x^2$. Set up but do not evaluate the integrals representing the volumes for the solids obtained by rotating T about:
   i) the y-axis
   ii) the line $y = 2$.

2. Find

$$\text{(i)} \quad \int \sqrt{x}\ln(x)\,dx; \qquad \text{(ii)} \quad \int \frac{(1-\sin(x))^2}{\cos^2(x)}\,dx.$$

3. Find $\displaystyle\int_0^1 \frac{dx}{\sqrt{2x - x^2}}$ or explain why it does not exist.

4. Find $\displaystyle\int_1^\infty \frac{dx}{x^2(x^2+1)}$ or explain why it does not exist.

5. A tank has the shape of a circular cone with its point downward and resting on the ground. The tank is four meters high and the diameter of the circle at the top is three meters. If the tank is filled to the depth of two meters with a liquid of density ρ find the work done in pumping the liquid over the top rim of the cone.

6. Determine whether or not the limit exists. if the limit exists find it and indicate clearly how you obtain your answer. If the limit does not exist give reasons why.

$$\text{(i)} \quad \lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln(x)}\right); \qquad \text{(ii)} \quad \lim_{x\to\infty}\left(\frac{2x+3}{2x+5}\right)^{3x}.$$

7. Determine whether or not each sequence of numbers defined below converges. If it converges find its limit. Justify your answer by citing appropriate results.

$$\text{(i)} \quad a_n = 1 + \frac{1}{e} + \frac{1}{e^2} + \cdots + \frac{1}{e^n}; \qquad \text{(ii)} \quad b_n = \frac{n\cos(n)}{2^n}.$$

8. (i) Show that the sequence $\{\sqrt{n+1} - \sqrt{n}\}$ is decreasing and converges to 0.
   (ii) Does the series $\sum(\sqrt{n+1} - \sqrt{n})$ converge or diverge? Justify your answer.
   (iii) Does the series $\sum(-1)^n(\sqrt{n+1} - \sqrt{n})$ converge or diverge? Justify your answer.

9. Find the radius of convergence *and* the interval of convergence of the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{n2^n x^n}{n^4 + 1}.$$

Find the Maclaurin series (Taylor series about 0) for $f''(x)$ and find its *interval* of convergence.

10. (i) Write down the Maclaurin series (Taylor series about 0) for $g(x) = e^{-x^2}$.
(ii) Use this series to evaluate the twentieth derivative $g^{(20)}(0)$.
(ii) Determine how many terms of this series should be used in order to approximate

$$\int_0^1 e^{-x^2}\, dx$$

with an error of at most 10^{-4}.