INSTRUCTIONS

Answer all questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. The exam is worth a total of 200 points.

1. [5 points each] Evaluate the limits, if they exist.

(a) $\lim\limits_{x\to 2} \dfrac{x^2+2x-8}{x-2}$

(b) $\lim\limits_{x\to 2^-} \dfrac{x^2+2x-5}{x-2}$

(c) $\lim\limits_{x\to 0} \dfrac{\sin x \tan x}{2x^2}$

(d) $\lim\limits_{x\to\infty} (\sqrt{x^2+x}-x)$

2. [6 points each] Find the derivatives of the following functions.

(a) $f(x) = e^{\sqrt{x}+7}, \quad x > 0.$

(b) $f(x) = \ln(x(2+\sin x)), \quad x > 0.$

(c) $f(x) = \displaystyle\int_2^{x^2} \cos u^2 \, du$

(d) $f(x) = (1+x^2)^x.$

3. [6 points each] Evaluate the following integrals, giving your answers in as simple a form as possible.

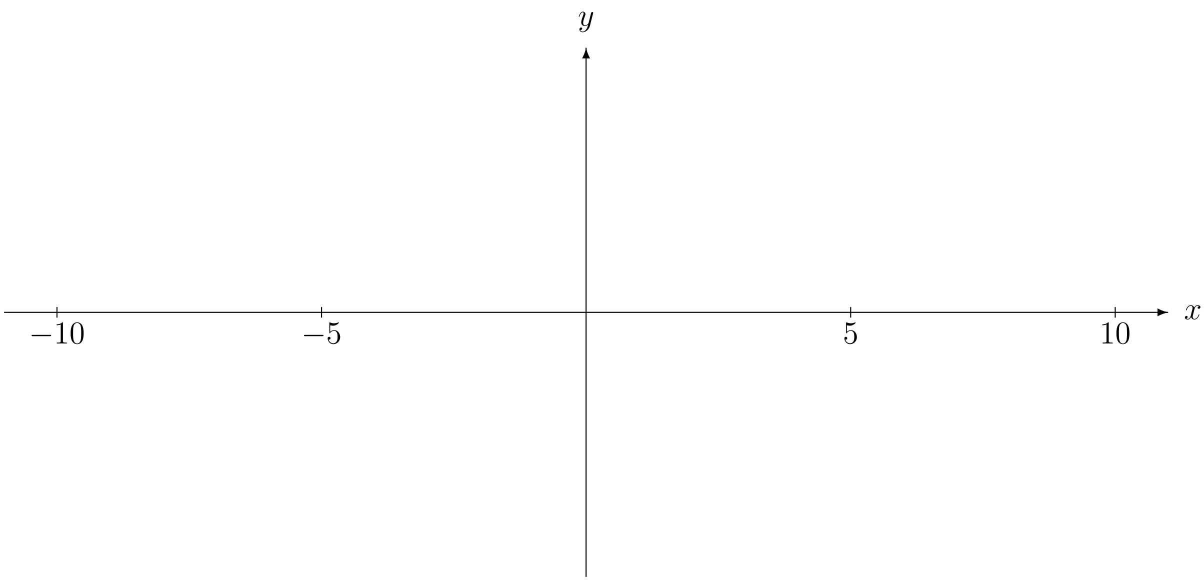
(a) $\displaystyle\int \frac{x^2 + 2\sqrt{x}}{x}\, dx$

(b) $\displaystyle\int_0^{\sqrt{\ln 18}} x e^{x^2}\, dx$

(c) $\displaystyle\int \frac{x}{1+x}\, dx$

(d) $\displaystyle\int_0^{\pi/2} \sqrt{\cos x}\sin x\, dx$

4. [30 points] Sketch the graph of the function $f(x) = \dfrac{x}{(x+3)^2}$ using the axes provided and fill in the required information below. If none, write NONE.



(i) Horizontal asymptotes (if any):. . . . .

(ii) Vertical asymptotes (if any):. . . . .

(iii) Positions of local maxima (if any):. . . . .

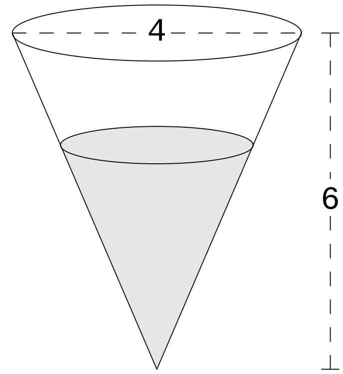
(iv) Positions of local minima (if any):. . . . .

(v) Positions of inflection points (if any):. . . . .

(vi) Intervals where f is concave upward (if any): . . . .

(vii) Intervals where f is concave downward (if any): . . . .

5. [20 points] Water is leaking from the bottom of a conical cup which is 4 inches in diameter and 6 inches deep. If the water leaks from the bottom at a rate of 0.3 cubic inches per minute, how fast is the water level dropping when the water is 5 inches deep?



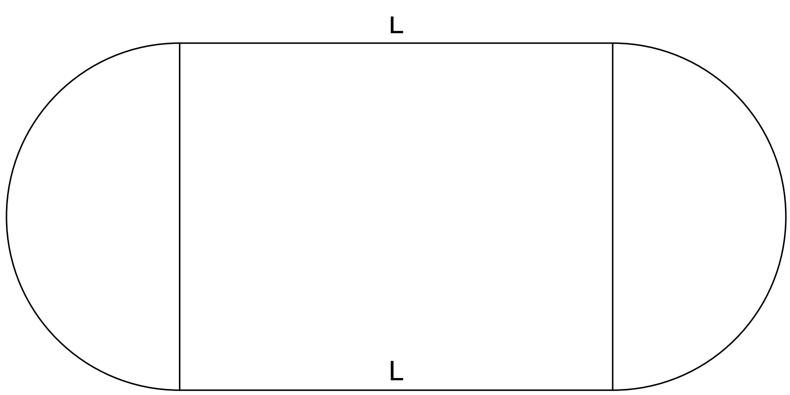
6. [15 points] Consider a curve given by the equation

$$(x^2 + 1)y^2 = xy^3 + 16.$$

(i) Find an equation of the tangent line to the curve at the point $P(-1, 2)$.

(ii) Suppose the point Q on the curve near P has x-coordinate -0.9. Use a linear approximation to estimate the y-coordinate of Q, giving your answer as a decimal.

7. [24 points] An oval running track is to be built by adding semicircles onto each end of a rectangle, as shown. The total perimeter of the track must be 400 meters, and the length L of each of the straight segments must be between 90 and 120 meters.

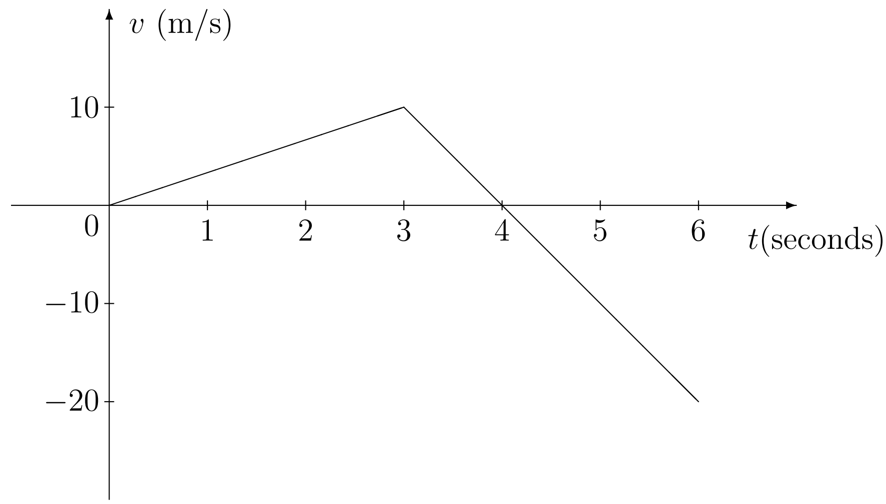


Find the value of L which will

(i) maximize the area of the **rectangle**;

(ii) minimize the area of the **rectangle**.

8. [15 points] Shown below is the graph of the velocity of a toy rocket that is launched into the air, uses up all its fuel, and falls back to the ground.



(i) When does the rocket reach its highest point in the air?

(ii) How high is it at that point in time?

9. [4 points each] True or false? If false give a counterexample. For this question only, you need not justify the answer 'true'.

(a) If f is continuous on some interval $[a, b]$, then f is differentiable on (a, b).

(b) If f is continuous on some interval $[a, b]$, then f has an antiderivative on $[a, b]$.

(c) If f and g are differentiable on the interval $(1, 2)$ and $f'(x) = g'(x)$ for all x in $(1, 2)$, then $f(x) = g(x)$ on $(1, 2)$.

(d) If $\lim_{x\to 0} f(x) = 3$, then there exists a number δ such that $|f(x) - 3| < 0.2$ whenever $0 < |x| < \delta$.

(e) If f is differentiable on the whole real line and $f'(c) = 0$, then f has a local maximum or a local minimum at $x = c$.

(f) If a function f which is differentiable on the whole real line has a local maximum or a local minimum at $x = c$, then $f'(c) = 0$.

(g) If $f(0) = f(1) = 5$ and f is continuous on $[0, 1]$ and differentiable on $(0, 1)$, then f has an absolute (global) maximum at some point $x = c$ in $[0, 1]$.