Math 126 Final Examination, Fall 2002

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1. Find the volume of the solid obtained when the region bounded by the curves $y = x^2$ and $x = y^2$ is rotated about the y-axis. Express the volume as an integral and evaluate it.

2. Evaluate the integral $\displaystyle\int \frac{3}{(x-1)(x^2+x+1)}dx$.

3. Consider the integral $\displaystyle\int_1^\infty \frac{(1/2)\sqrt{x} + (2/3)\sqrt[3]{x}}{x^2}\,dx$. Determine whether it converges. If so, find the value of the integral.

4. The curve $y = \cosh x, \ -1 \le x \le 1$, is rotated about the x-axis. Find the area of the resulting surface:

   (a) Express the area as an integral.

   (b) Evaluate the integral for the area. Express the final answer in terms of a hyperbolic function.

5. The end of an open tank containing water is vertical and is a semicircle with diameter 16 meters at the top. The surface of the water is 4 meters from the top. Set up an integral for the hydrostatic force exerted against the end of the tank. Do not evaluate the integral. You may leave your answer in terms of the density of water (ρ) and the acceleration due to gravity (g).

6. Consider the curve $r = 3 + 2\cos\theta$.

   (a) Sketch the curve.

   (b) Find the area enclosed by the curve. (Express the area as an integral and evaluate it.)

7. Determine if the series is absolutely convergent, conditionally convergent, or divergent. Clearly state any test you use and show that all of the necessary conditions for applying the test are satisfied.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

   (b) $\displaystyle\sum_{n=2}^{\infty} (1 - \frac{1}{n})^{-n}$

8. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n2^n}(x-3)^n$.

   (a) Find the radius of convergence of the power series.

   (b) Find the interval of convergence of the power series.

9. Let $f(x) = e^{x^2}$.

   (a) Write the Maclaurin series for $f(x)$.

   (b) Compute $f^{(10)}(0)$, the 10^{th} derivative of $f(x)$ evaluated at 0.

10. How many nonzero terms of the Maclaurin series for $\ln(1+x)$ do you need to estimate $\ln 1.4$ to within 0.01? Be sure to justify your solution.