Math 125, Spring 2002, Calculus I
PRACTICE FINAL EXAM

(This was the final exam for Math 125 in the Fall 2001 semester)

Instructions: Try all the problems and show all your work. Answers given with no indication of how they were obtained may receive no credit.

If you need more space, write on the back of another page and clearly mark on your problem that it is continued elsewhere.

Problem 1. (45 points) Find the limits.

a. $\lim\limits_{x\to 4} \dfrac{x^2-5x+4}{x-4}$

b. $\lim\limits_{x\to -\infty} \dfrac{\sqrt{x^2+1}}{2x-5}$

c. $\lim\limits_{x\to 0} \dfrac{\sin 3x}{x+\sin 4x}$

Problem 2. (15 points) Find a constant c for which the following function f is continuous on $(-\infty,\infty)$.

$$f(x)=\begin{cases} 2x^2+cx & \text{if } x\ge 1\\ x^2-2x-c & \text{if } x<1\end{cases}$$

Problem 3. (30 points) Find dy/dx if

a. $y=\dfrac{e^x}{\sin x}$ b. $y=\ln(\sec x+\tan x)$. Simplify the result.

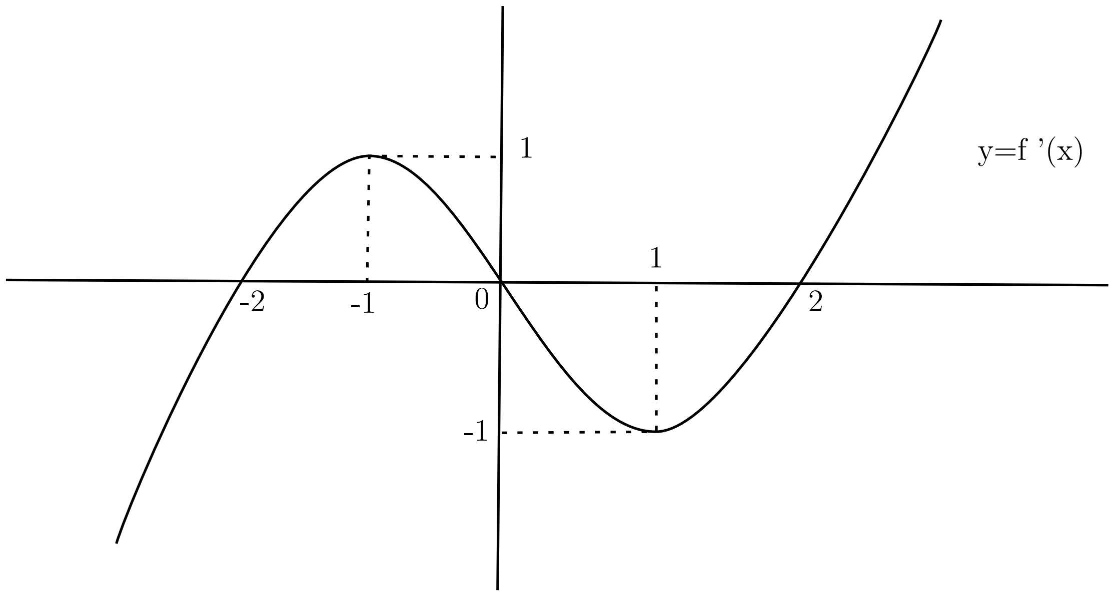
Problem 4. (60 points) Consider the function $f(x)=3x^5-5x^3$.

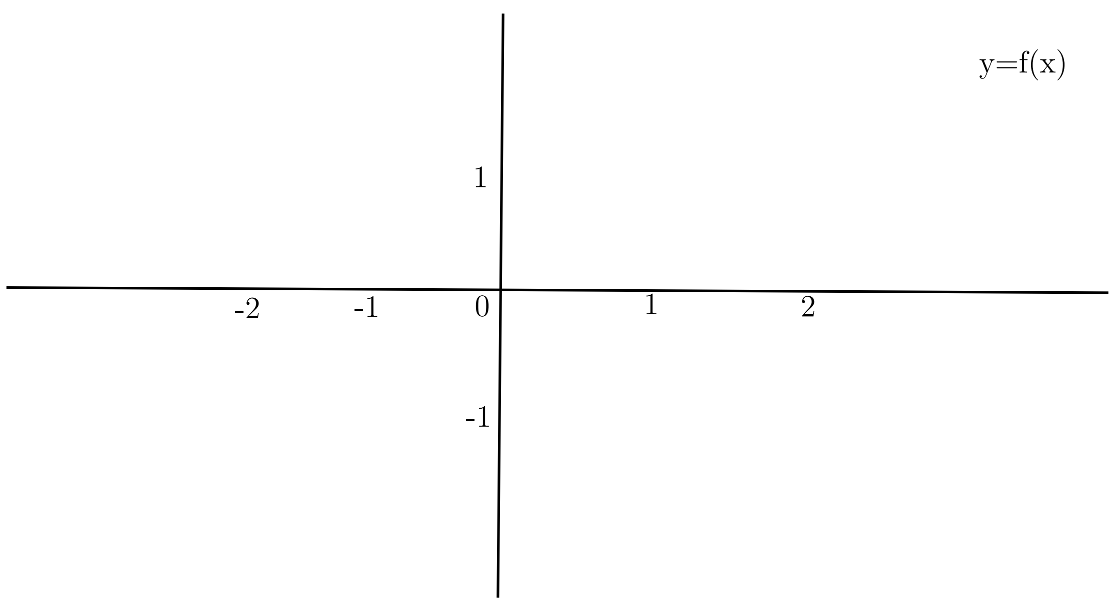
Sketch a graph of the function f, and indicate on the dotted lines below:

(1) the intervals where the function is increasing (if any): ....
(2) the intervals where the function is decreasing (if any): ....
(3) the x–coordinates of local maxima (if any): ....
(4) the x–coordinates of local minima (if any): ....
(5) the intervals where the function is concave up (if any): ....
(6) the intervals where the function is concave down (if any): ....
(7) the x–coordinates of inflection points (if any): ....

Problem 5. (30 points) Let the graph below be the graph of the DERIVATIVE $f'(x)$ of a function $f(x)$ for which $f(0)=0$. Sketch a possible graph for the function $f(x)$, and indicate on the dotted lines below:

(1) the x–coordinates of local maxima of the function $f(x)$ (if any): ....
(2) the x–coordinates of local minima of the function $f(x)$ (if any): ....
(3) the x–coordinates of inflection points of the function $f(x)$ (if any): ....



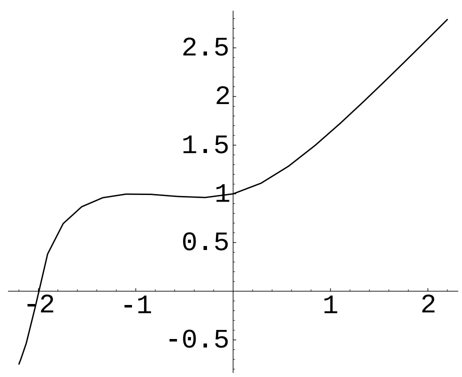


Problem 6. (40 points) 10,000 cubic meters of oil is spilled into the ocean. The oil takes the shape of a cylinder. The radius of the cylinder increases at a rate of 4 meters/hour. At

what rate is the thickness decreasing when the radius is 100 meters? (We assume that the oil and the water are not mixing, so that the volume of the cylinder remains constant.)

Problem 7. (40 points) A storage tank has the shape of a cylinder with ends capped by two flat disks. The price of the top and bottom caps is \$ 3 per square meter. The price of the cylindrical walls is \$ 2 per square meter. What are the dimensions of the cheapest storage tank that has a volume of 1 cubic meter?

Problem 8. (25 points) Consider the curve described by the equation $y^3+y-2=x(x+1)^2$, whose graph is shown below.



a. Find the slope of the line tangent to the curve at $(0,1)$.

b. Use linear approximation to estimate the y–coordinate of the point on the curve whose x–coordinate is equal to 0.1.

c. Is your estimate in Part b an overestimate or an underestimate, namely is your estimate more or less than the actual value of the y–coordinate? Explain, using the graph.

Problem 9. (15 points) Compute the derivative of the function

$$f(x)=\int_0^{5x} t\sqrt{t+2}\,dt$$

Problem 10. (30 points) Evaluate the following indefinite integrals:

a. $\int \sqrt{3-2x}\,dx$

b. $\int \frac{dx}{x\ln x}$

Problem 11. (40 points) Evaluate the following definite integrals:

a. $\displaystyle\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}}\,dx$

b. $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx$

Problem 12. (30 points) A train leaves a station and, t minutes after departure, its velocity in miles/minute is given by the formula

$$v(t) = \begin{cases} 0.1\,t & \text{if } 0 \le t \le 10 \\ 3 - 0.2\,t & \text{if } 10 \le t \le 15 \end{cases}$$

How far did the train travel in these 15 minutes?