Math 126 - Final - Fall 2001

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1. (15 points) SET UP the integral to find the area of the "sunglasses" formed by the graphs of $y = 2x - x^3$ and $y = x^3$.

   Do Not Evaluate Integrals.

2. (15 points) A hemispherical tank of diameter 10 feet at the top is filled with water 1 foot from the top. SET UP the integral to find the work required to pump enough water out over the top so that the level drops 1 foot. The weight density of water is 62.5 lb/ft^3. Do not evaluate integrals or carry out multiplications of numbers.

3. (15 points) Integrate $\int x^3 e^{x^2} dx$.

4. (10 points) Find the limit or say that it does not exist: $\lim_{x \to 1^+} (x^2 + x - 1)^{\frac{1}{x-1}}$.

5. (15 points) Integrate $\int \frac{x^2}{\sqrt{1-x^2}} dx$.

6. (20 points) Integrate $\int \frac{x^2 + x + 1}{x^3 + x} dx$.

7. (15 points) A tank has ends in the shape of the region between the curves $y = \frac{1}{3}x^2$ and $y = 6$ with distance measured in feet. SET UP the integral for the hydrostatic force on one end of the tank when it is filled to a depth of 3 feet with water. The weight density of water is 62.5 lb/ft^3. Do not evaluate integrals or carry out multiplications of numbers.

8. (15 points) Consider the curve $r = 3\sqrt{1 + \cos\theta}$.

   (a) Sketch the curve.

   (b) Find the area that it encloses.

9. (15 points) Let T be the region below the x-axis and bounded by the y-axis and the graph of $y = ln(x)$. If T is rotated about the y-axis, does the resulting solid have a finite volume? Find the volume or explain why the volume is not finite.

10. (20 points) Determine convergence or divergence for the following two series. State the test that you use, and verify that all necessary conditions for applying the test are satisfied.

    (a) $\sum_{n=1}^{\infty} \frac{3^n + 4^n}{5^n}$ (b) $\sum_{n=1}^{\infty} \frac{2n}{\sqrt{2 + n^3}}$

11. (15 points) Find the radius and interval of convergence for the power series $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} 2^n$.

12. (15 points) Consider the Taylor series expansion of the function $f(x) = x\, ln\, x$ about $x = 1$. Give the first 4 non-zero terms.

13. (15 points) (a) Write down the Taylor series for e^{-x^2}.

    (b) Use this series to approximate $\int_0^{0.1} e^{-x^2} dx$ with an error of at most 10^{-4}.