Math 118, Spring 2022 **Final** Name: ____________________

Instructions:

- Write your name in the space indicated above.
- You have 2 hours to complete your work. You can use a handwritten sheet of notes (8.5x11, front and back) and a non-graphing calculator as you solve these problems.
- Make sure to clearly show each step of your solution for full credit - correct answers without any work will not receive credit. Some questions may remind you to show your work explicitly, but you should assume each question requires you to justify your steps by making your thought process clear in your solution.
- All work you submit should represent your own thoughts and ideas. If the graders suspect otherwise, you can expect your instructor to file a report with USC Student Judicial Affairs and Community Standards.
- Indicate your section by circling the instructor and time below.

Warner, 9am   Haskell, 12pm   Haskell, 1pm   Warner, 1pm   Zhang, 2pm

Question	**Score**
1	/15
2	/10
3	/15
4	/15
5	/15
6	/15
7	/15
8	/15
9	/10
10	/25
Total	/150

1. (15 points) A theater is currently selling tickets for \$80 and at this price, they sell 130 tickets each night. They estimate that for each \$5 increase in the price, they will sell 10 fewer tickets.

   (a) Find the elasticity of demand at the current price.

   (b) If the theater increases ticket price, will revenue increase or decrease? Explain.

   (c) At what price should they sell their tickets in order to maximize their revenue?

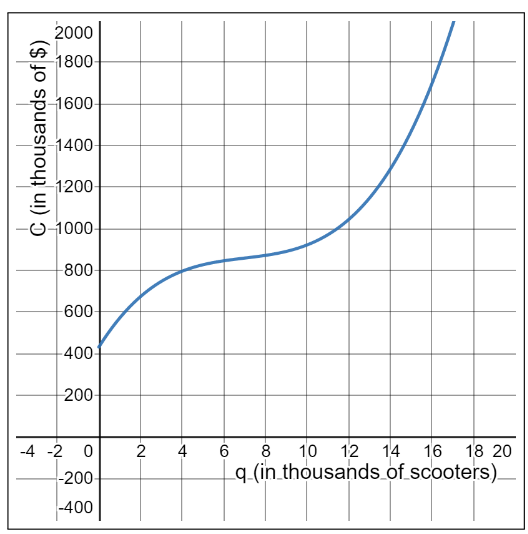
2. (10 points) A company sells electric scooters. Their profit (in thousands of dollars) is

$$\Pi(q) = -q^3 + 21q^2 - 39q - 432$$

where q is measured in thousands of scooters. Due to a shortage in raw materials, the manufacturer is only able to produce at most 12 thousand scooters.

(a) Find the production quantities at which the manufacturer achieves a maximum and minimum profit. You must demonstrate these quantities are actually the maximum and minimum.

(b) A graph of the manufacturer's total cost function is given to the right. Use the graph to estimate the quantity at which the manufacturer's average cost per item is minimized. Show graphically how you obtained your answer.



3. (15 points) Pollutants in a factory are removed by air filters that become less efficient as time goes on. The following measurements, made twice each month, show the rate at which pollutants are removed.

Time, t (months)	0	0.5	1	1.5	2	2.5	3
Rate of removal, r (tons/month)	6.0	4.0	2.8	2.0	1.4	1.0	0.7

(a) Use the table to identify the sign (positive or negative) of $r''(t)$. Explain your answer.

(b) Suppose we know that $r'(1.5) = -1.5$. Use this to estimate $r(1.2)$.

(c) Is your answer from part (b) an upper estimate or lower estimate? Explain.

(d) Find upper and lower estimates of the total amount of pollutant that is removed in the six month period.

4. (15 points) The total cost C (in dollars) to drill an oil well consists of fixed costs of \$350,000 (independent of the depth of the well) and marginal costs, which depend on the depth of the well. (After a certain depth, drilling generally becomes more expensive, per meter, the deeper the well is drilled.) The marginal costs at a depth of x meters are

$$\mathrm{MC}(x) = \frac{10,000}{10 + x} + 3x$$

dollars per meter.

(a) If the well has already been drilled to a depth of 100 meters, approximately how much does it cost to drill another meter?

(b) If the well has already been drilled to a depth of 100 meters, how much does it cost to drill another 100 meters?

This problem continues on the next page.

(c) Find the total cost C to drill to a depth of x meters.

5. (15 points) Evaluate the following definite and indefinite integrals.

(a) $\displaystyle\int_0^3 x\sqrt{x^2+16}\,dx$

(b) $\displaystyle\int \left(\sqrt{x} - e^{2x} + \frac{1}{5x}\right)\,dx$

6. (15 points) The supply and demand curves for rib-eye steaks in the small town of Agraville are

$$p_s(q) = q + 6 \qquad \text{and} \qquad p_d(q) = \frac{50}{q+1}$$

where q is the quantity (in thousands of pounds), $p_s(q)$ is the selling price (in dollars) at which suppliers will supply that quantity, and $p_d(q)$ is the selling price (in dollars) at which consumers will purchase that quantity.

(a) Find the equilibrium price and quantity.

(b) Find the consumer surplus at equilibrium. Include units in your answer.

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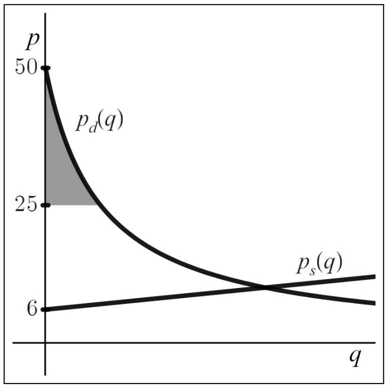
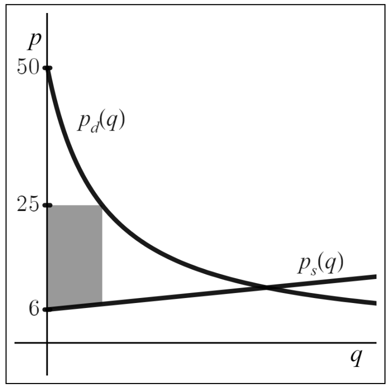
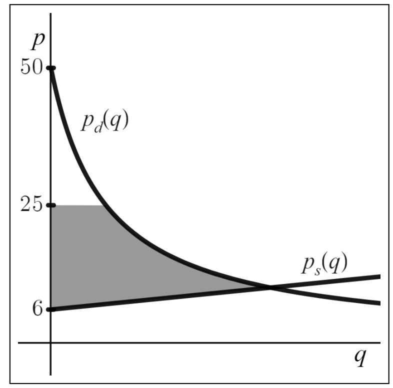
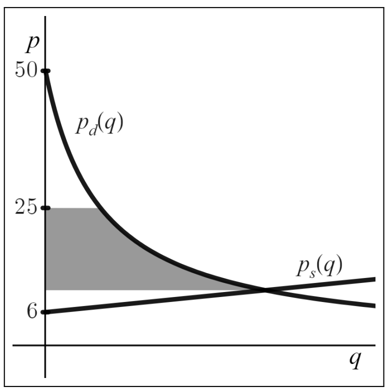
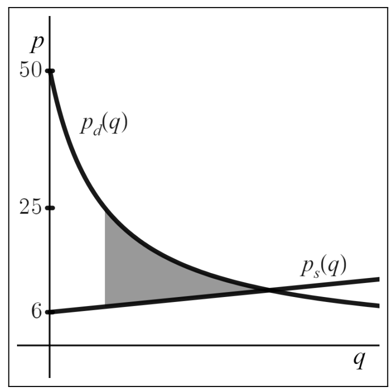
(c) Suppose the price of a rib-eye is set at \$25. Circle the graph below whose shaded region represents the producer surplus when $p = \$25$.











(d) Compute the value of the producer surplus when $p = \$25$. Include units in your answer.

7. (15 points) A large company purchases a start-up company for 30 million dollars. The start-up company is expected to generate an income stream of $S(t) = 3e^{0.2t}$ million dollars per year, where t is the number of years since the purchase.

   (a) How much total income does the start-up company generate within the first five years after its purchase?

Suppose the prevailing interest rate is 10% compounded continuously.

   (b) What is the present value of the income stream over the first five years after the start-up company's purchase?

This problem continues on the next page.

(c) How long will it take for the purchase of the start-up company to pay for itself? In other words, over what period of time will the present value of $S(t)$ equal the cost to purchase the start-up?

8. (15 points) A family-run business makes cowboy boots. The number of boots they can make each month Q, depends on the amount of labor they employ L, (measured in hundreds of people-hours) and the amount they invest in equipment, K (measured in thousands of dollars), as shown in the table to the right.

		Capital (K)				
		0	4	8	12	16
Labor (L)	0	0	128	256	384	512
	1	4	291	500	695	881
	2	8	349	582	796	1000
	3	12	394	645	873	1089
	4	16	432	698	938	1164

(a) Estimate $\dfrac{\partial Q}{\partial K}$ at $(L, K) = (1, 8)$. Include units in your answer.

(b) Which of the following are possible interpretations of your answer from part (a)? Circle all that apply.

i. The approximate number of extra boots the company will make if they invest in another one hundred people-hours per week.

ii. The approximate number of extra boots the company will make if they invest in another one thousand dollars of equipment.

iii. The approximate number of extra boots the company will make if they invest in another one hundred people-hours and another 8 thousand dollars of equipment.

iv. The rate at which the number of boots the company makes is increasing per month.

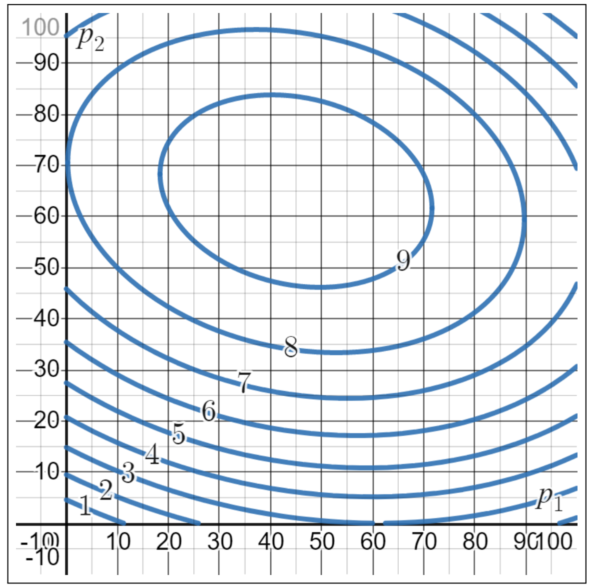
v. The rate at which the number of boots the company makes is changing when it employs another one hundred people-hours and invests in another 8 thousand dollars of equipment.

This problem continues on the next page.

(c) Suppose we now know that the formula for this production function is $Q(L, K) = 4\left(L^{\frac{1}{3}} + 2K^{\frac{1}{3}}\right)^{3}$. Use this formula to find the exact value of $\dfrac{\partial Q}{\partial L}$ at $(L, K) = (1, 8)$.

(d) Use linear approximation to estimate $Q(1.2, 7.7)$. You must use linear approximation to receive any credit. You may use your estimate from part (a) and the exact value from part (c) in your approximation.

9. (10 points) A company sells black ink cartridges at a price of p_1 dollars per cartridge and color ink cartridges at a price of p_2 dollars per cartridge. A contour diagram for the total revenue $R(p_1, p_2)$ (in millions of dollars) is shown to the right.



(a) Estimate a price pair (p_1, p_2) that maximizes revenue. Illustrate your estimate on the diagram and briefly explain your reasoning.

(b) Estimate a price pair (p_1, p_2) that maximizes revenue if the company wants to be able to sell a black cartridge and a color cartridge together for a total of \$60. Illustrate your answer on the diagram and briefly explain your reasoning.

(c) The Lagrange multiplier for the constrained optimization problem in part (b) is 0.062. Use this to estimate the change in maximum revenue if the company now decides to sell both cartridges together for \$63.

10. (25 points) A restaurant has determined that the number of orders of burgers per hour, q_1, and the number of orders of fries per hour, q_2, are related to the price of a burger, p_1 dollars, and the price of fries, p_2 dollars, as follows:

$$q_1 = 100 - 3p_1 - 7p_2$$

$$q_2 = 140 - p_1 - 6p_2$$

(a) Find a formula for $R(p_1, p_2)$, the hourly revenue the restaurant generates as a function of p_1 and p_2.

(b) Use your formula from part (a) to find a price pair (p_1, p_2) that corresponds to a local maximum for the revenue function. Be sure to justify that your answer is a local maximum.

This problem continues on the next page.

(c) What price pair (p_1, p_2) maximizes revenue if the restaurant wants to be able to sell a burger and fries together for \$14?