Mathematica-Compatible Notebook

This notebook can be used on any computer system with Mathematica 3.0, MathReader 3.0, or any compatible application. The data for the notebook starts with the line of stars above.

To get the notebook into a Mathematica-compatible application, do one of the following:

* Save the data starting with the line of stars above into a file with a name ending in .nb, then open the file inside the application;

* Copy the data starting with the line of stars above to the clipboard, then use the Paste menu command inside the application.

Data for notebooks contains only printable 7-bit ASCII and can be sent directly in email or through ftp in text mode. Newlines can be CR, LF or CRLF (Unix, Macintosh or MS-DOS style).

NOTE: If you modify the data for this notebook not in a Mathematica-compatible application, you must delete the line below containing the word CacheID, otherwise Mathematica-compatible applications may try to use invalid cache data.

For more information on notebooks and Mathematica-compatible applications, contact Wolfram Research:
  web: http://www.wolfram.com
  email: info@wolfram.com
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Notebook reader applications are available free of charge from Wolfram Research.
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*******************************************************************)

(*CacheID: 232*)


(*NotebookFileLineBreakTest
NotebookFileLineBreakTest*)
(*NotebookOptionsPosition[     32141,       1021]*)
(*NotebookOutlinePosition[     33287,       1057]*)
(*  CellTagsIndexPosition[     33243,       1053]*)
(*WindowFrame->Normal*)



Notebook[{

Cell[CellGroupData[{
Cell[TextData[{
  "A Gentle Introduction\nto ",
  StyleBox["Mathematica ",
    FontSlant->"Italic"],
  "II"
}], "Title",
  Evaluatable->False,
  AspectRatioFixed->True],
```

```
Cell["\<\
If you think this lesson looks pretty bare, remember:  you have to \
double-click the barbs to the right of a section before you can see what's in
\
the section!\
\>", "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[CellGroupData[{

Cell[TextData[{
  "Assigning Values",
  StyleBox["",
    FontSlant->"Italic"]
}], "Section",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[TextData[{
  "When you place the cursor in an active ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " cell and press the Enter key, ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " evaluates the expression and returns the value in the",
  StyleBox[" Out[]",
    FontFamily->"Courier"],
  " value.  Try that in the following line:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(x\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "You ",
  StyleBox["should ",
    FontSlant->"Italic"],
  "have got just \"x\" as the answer.  (If you didn't get \"x\", you'll \
understand why in a few moments.)\n\nThat's because ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " doesn't know anything more about \"x\" than its name.  But you can assign
\
a value to x, which ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " will remember in later calculations.  For example:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],
```

```
Cell[BoxData[
    \(x = 5\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"Now let's try an expression which involves x.  As usual, move the cursor to \
\
the following line and hit the Enter key:"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(x\^2\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "We could type more complicated expressions, but the idea is the same:  ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " will treat the symbol x just as if it were a 5.\n\nSometimes this can be \
\
annoying; \"x\" is too nice a name for an unknown quantity to have ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " keep treating it as 5. There's a command to make ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " forget the value that x has:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"Now when we type an expression involving x, we're sure that it isn't \
carrying around a value we forgot about.  Example:"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(x\^2\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "As long as you clear the value of x before using it, you're safe.  ",
  StyleBox[
  "It's a good idea to Clear all the variables you're going to use, before \
you use them.",
    FontWeight->"Bold"]
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True]
}, Closed]],
```

```
Cell[CellGroupData[{

Cell[TextData["Mathematica Solves Equations"], "Section",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[TextData[{
  "One of the nicest features of ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " is its ability to solve equations.  You surely remember how to solve \
quadratic equations, so before pressing \"Enter\" in the next cell, solve it 
\
by hand:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; Solve[x\^2 - 3\ x + 2 == 0, x]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "It's pretty clear what we're asking ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " to do; we've asking it to solve the quadratic equation x^2 -3x +2 = 0 for 
\
x.  The only thing that might not be clear is, Why the DOUBLE equal sign?  \
That's necessary because we've already coopted the equal sign for assignment 
\
(remember the last section); so we have to invent a new symbol for \
\"equals\".  (The same problem exists in the programming language C, which is 
\
where ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " got the idea.)\n\nNotice that the answer is a LIST (things inside curly \
braces) of RULES (x -> 1 means \"assign x the value 1\").  Why do it this \
way?  Why not just return a list of solutions?  That was just part of the \
design of ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " (and this way of doing things has some advantages, as we'll see).\n\n\
Note, in particular, that ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " doesn't change the value of x.  It's left untouched.\n\nAlso notice how \
important the Clear[x] is!!  Suppose we wrote instead"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(x = 5; Solve[x\^2 - 3\ x + 2 == 0, x]\)], "Input",
  AspectRatioFixed->True],
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Cell[TextData[{
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " can't solve it!!  Of course not; you've remade \"x\" into \"5\", and ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " knows that \"5\" is not a variable.  It doesn't make any sense to solve \
an equation for \"5\".\n\n",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " prefers to give exact answers, as we see in an equation which doesn't \
have such nice solutions:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; Solve[x\^2 - x - 1 == 0, x]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"If we're looking for numerical answers, we can get them to any precision; \
for example,"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; N[Solve[x\^2 - x - 1 == 0, x], 10]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "This gives very precise answers.  Incidentally, ask ",
  StyleBox["Mathematica\n?",
    FontSlant->"Italic"],
  StyleBox["GoldenRatio ",
    FontFamily->"Courier"],
  "sometime.\n\nNow you probably wouldn't buy a $1195 program just to solve \
quadratic equations.  ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " can solve cubics, too:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; Solve[x\^3 + x + 1 == 0, x]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"Gulp!  Well, it might be easier to see the answers if we display them \
numerically:"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(N[%]\)], "Input",
```

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  AspectRatioFixed->True],

Cell[TextData[{
  "Now that last line needs to be executed IMMEDIATELY after the line which \
solved the cubic equation.  That's because ",
  StyleBox["%",
    FontFamily->"Courier"],
  " always refers to the result of the last calculation, and if you wandered 
\
off somewhere else in the worksheet and did some other calculation in the \
meantime, the last result wouldn't be the solution of the cubic.  ",
  StyleBox["%%",
    FontFamily->"Courier"],
  " refers to the result of the calculation done just prior to the last \
calculation;",
  StyleBox[" %%%",
    FontFamily->"Courier"],
  " to the one just before that; etc.  (You could also use",
  StyleBox[" %n",
    FontFamily->"Courier"],
  ", where ",
  StyleBox["n",
    FontFamily->"Courier"],
  " is the number of the",
  StyleBox[" Out[]",
    FontFamily->"Courier"],
  " result which ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " returned.  I can't do that here, because I don't know what number was \
assigned to the result of calculating the cubic, since I can't be sure you \
did all the examples in the correct order.)\n\nNotice that two of the roots \
of that last equation were complex numbers.  Remember that \"I\" is the \
square root of \[Dash]1 (NOT i).\n\n",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " can also solves quartic equations, that is, fourth-degree polynomial \
equations.  I encourage you to experiment on your own; the answers will \
probably go on for pages.  It will not usually solve quintics (5th degree) or 
\
higher polynomial equations, nor will it usually factor polynomials of degree 
\
5 or higher (unless you're lucky and they have simple roots), because of a \
famous theorem in mathematics which says such equations ",
  StyleBox["cannot",
    FontSlant->"Italic"],
  " be solved (except for special cases) using a finite number of the \
arithmetic operations of +, \[Dash], *, /, and taking n-th roots.\n\nHowever, 
\
if you'd be satisfied with numerical approximations to roots of polynomial \
equations\[LongDash]the situation most of you will usually be \
in\[LongDash]then ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " has a special function to return those:"
}], "Text",
  Evaluatable->False,
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  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; NSolve[x\^3 + x + 1 == 0, x]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "This has almost the same effect as ",
  StyleBox["N[Solve[x^3+x+1==0,x]], ",
    FontFamily->"Courier"],
  "except the latter may be somewhat more accurate.  But ",
  StyleBox["NSolve ",
    FontFamily->"Courier"],
  "uses numerical algorithms, and will return numerical approximations when
",

  StyleBox["Solve ",
    FontFamily->"Courier"],
  "just gives up.  Try"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Solve[x\^6 + x + 1 == 0, x]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData["and then try"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(NSolve[x\^6 + x + 1 == 0, x]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData["or"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(NSolve[x\^6 + x + 1 == 0, x, 30]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "\nA variation on ",
  StyleBox["NSolve ",
    FontFamily->"Courier"],
  "is",
  StyleBox[" NRoots.",
    FontFamily->"Courier"],
  " Ask ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " about it; it returns the roots in a different way.",
  StyleBox["\n\n",
    FontFamily->"Courier"],
  "However, ",
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  StyleBox["NSolve",
    FontFamily->"Courier"],
  " and ",
  StyleBox["NRoots",
    FontFamily->"Courier"],
  "  only work for ",
  StyleBox["polynomial",
    FontSlant->"Italic"],
  " equations.  We can't use them to try to solve, say, sin(x) = x*cos(x).  \
There is a procedure, ",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  ", which is tailored for such transcendental equations, but it requires \
making a shrewd guess at what the root is.  Here's one way we can do 
this:\n\n\
Let's begin by graphing the functions sin(x) and x*cos(x) on the same plot.
\
The points where the graphs cross will yield the solutions of the equation.
\
Let's try this between x = 0 and x = 10:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Plot[{Sin[x], x\ Cos[x]}, {x, 0, 10}]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"It looks like there are three crossings in the picture; at x = 0, somewhere
\
between x = 4 and x = 5, and somewhere near x = 8.  Let's look for the root \
near 8:"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Plot[{Sin[x], x\ Cos[x]}, {x, 7, 8}]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "(When you're working by yourself, you might prefer to just change the \
\"0\" and \"10\" in the previous line to \"7\" and \"8\" respectively.  Saves
\
typing.  Or you can just copy the previous command and paste it into a new \
cell, then change the \"0\" and \"10\" in the copy; that has the advantage of
\
preserving the first graph for your later perusal\[LongDash]or your \
instructor's later perusal.)\n\nFrom this it looks like there's a root \
somewhere between 7.7 and 7.8.  We could keep zooming in on the intersection
\
point, getting better and better precision, but let's try ",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  ":"
}], "Text",
  Evaluatable->False,
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  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; FindRoot[Sin[x] == x\ Cos[x], {x, 7.7}]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "Remember, to find out the rules for using ",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  ", you type ?",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  ", hit Enter, and ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " summarizes them for you.  What the procedure wants is an equation, which 
\
is the first thing we gave it, together with a list which specifies the \
variable and a numerical guess as to what the root is.  We didn't need to be 
\
so precise about our guess:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; FindRoot[Sin[x] == x\ Cos[x], {x, 9}]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "As a working tool, ",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  " is very valuable, but it must be used with some caution.  It is a \
\"black-box\" procedure, one which purports to solve a very difficult problem 
\
(solving equations) with a minimum of user interaction.  In fact, what ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " has done here is use Newton's method to solve the equation, with a \
maximum of 15 iterations.  (Newton's method is covered in Math 125.)  \
Calculations were done to 16 digits of accuracy internally, but ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " stopped once it got the answer to five decimal places.  You can specify \
other options in ",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  "; if you want more digits of accuracy, try"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; \n
    FindRoot[Sin[x] == x\ Cos[x], {x, 9}, WorkingPrecision \[Rule] 50]\)],
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  "Input",
  AspectRatioFixed->True],

Cell[TextData[
"Remember the rules for getting help:  you can always type"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Options[FindRoot]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData["followed by"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(\(?AccuracyGoal\)\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"to find out what terms mean.\n\nFinally, let's solve the equation x^x = 
2:"],
   "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Clear[x]; \nFindRoot[x\^x == 2, {x, 1}, WorkingPrecision \[Rule] 
50]\)],
   "Input"],

Cell[TextData[
"There's no particular reason to do this, except that when I took high-school 
\
Algebra I, I asked the teacher how one would solve it (we had just finished \
quadratic equations), and she didn't have any idea."], "Text",
  Evaluatable->False,
  AspectRatioFixed->True]
}, Closed]],

Cell[CellGroupData[{

Cell[TextData["Mathematica Does Tables"], "Section",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[TextData[{
  "Graphing functions is a very useful way of visualizing the properties of \
the function.  The human eye and brain are able to absorb a great deal of \
information about shape, relative sizes, etc.  But graphs don't give very \
precise information about exact values.  Tables of values are traditionally \
used for this.\n\nConsider, for example, what happens to the ratio ",
  StyleBox["Sin[x]/x",
    FontFamily->"Courier"],
  " as x approaches zero.  One way we can estimate what happens to the ratio 
\
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is to compute some values of ",
  StyleBox["Sin[x]/x",
    FontFamily->"Courier"],
  " for small values of x.  It's quite easy:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(t = Table[N[Sin[1\/2\^n]\/\(1\/2\^n\)], {n, 1, 10}]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"This command creates a table (list) of the values of Sin[x]/x for x = 1/2, \
1/4, 1/8, \[Ellipsis], 1/1024.  Well, it sure looks like it's getting close \
to 1!  (Why did we put the N in there?  Try it without, then try to tell \
whether the result is approaching 1!)"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[TextData[{
  "That's a pretty ugly way of writing the table, though, because it's \
difficult to read, and even more difficult to compare values.  ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " has a way of making the display more attractive (it's called \
\"prettyprinting\").  One is to print the ",
  StyleBox["TableForm",
    FontFamily->"Courier"],
  " of the list:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(TableForm[t]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"Ahhh, much better!  And notice how clever we were to give the list a name \
(\"t\")\[LongDash]we didn't have to retype it all.  Here's another example of
\
a table, this time of numerical values of Sin:"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(Table[N[Sin[x]], {x, 0, \[Pi], \[Pi]\/10}]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "The ",
  StyleBox["{x,0,Pi,Pi/10}",
    FontFamily->"Courier"],
  " is called an \"iterator\"; it tells us how many times to perform the \
calculation, and for which values of x.  This one tells ",
  StyleBox["Mathematica",
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    FontSlant->"Italic"],
  " we want x to run from 0 to",
  StyleBox[" Pi",
    FontFamily->"Courier"],
  ", but in steps of",
  StyleBox[" Pi/10",
    FontFamily->"Courier"],
  ".  That last information is necessary for ",
  StyleBox["Table",
    FontFamily->"Courier"],
  ", since we have to know how many elements to generate for the list.\n\nWe \
\
can't use ",
  StyleBox["Plot",
    FontFamily->"Courier"],
  " to plot numbers in a list, because ",
  StyleBox["Plot",
    FontFamily->"Courier"],
  " expects a function.  Instead we use ",
  StyleBox["ListPlot",
    FontFamily->"Courier"],
  ".  Do you remember the Binomial Theorem?  Well,",
  StyleBox[" Binomial[n, k]",
    FontFamily->"Courier"],
  " is the coefficient of ",
  StyleBox["x^k",
    FontFamily->"Courier"],
  " in the expansion of",
  StyleBox[" (1+x)^n",
    FontFamily->"Courier"],
  ".  Let's plot these for n = 25:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(ListPlot[Table[Binomial[25, k], {k, 25}]]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "One of the options for ",
  StyleBox["ListPlot",
    FontFamily->"Courier"],
  " is",
  StyleBox[" PlotJoined->True",
    FontFamily->"Courier"],
  ", which makes the graph easier to see:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(ListPlot[Table[Binomial[25, k], {k, 25}], PlotJoined \[Rule] True]\)],
  "Input",
  AspectRatioFixed->True],

Cell[TextData[{
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  "Alternatively, if you just plot enough points, the graph may ",
  StyleBox["look ",
    FontSlant->"Italic"],
  "like a solid line:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(ListPlot[Table[Prime[n], {n, 1, 1000}]]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[{
  "You don't have to ",
  StyleBox["guess",
    FontSlant->"Italic"],
  " what",
  StyleBox[" Prime[n]",
    FontFamily->"Courier"],
  " means; ",
  StyleBox["ask Mathematica!\n\n",
    FontSlant->"Italic"],
  "The plot looks pretty regular, right?  Look again!   We'll take a smaller \
\
piece of the graph:"
}], "Text",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[BoxData[
    \(ListPlot[Table[Prime[n], {n, 500, 600}]]\)], "Input",
  AspectRatioFixed->True],

Cell[TextData[
"If you click on that last plot, thereby selecting it, and then go up to the \
\
\"Graph\" menu and select \"Make Lines Thin\", the points in the graph won't \
\
be so thick, and it will be easier to distinguish individual points.  There's \
\
nothing very regular about prime numbers!"], "Text",
  Evaluatable->False,
  AspectRatioFixed->True]
}, Closed]],

Cell[CellGroupData[{

Cell[TextData["Review"], "Section",
  Evaluatable->False,
  AspectRatioFixed->True],

Cell[TextData[{
  "You've learned how to ",
  StyleBox["assign values ",
    FontWeight->"Bold"],
  "to variables.  These values can be any ",
  StyleBox["Mathematica",
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    FontSlant->"Italic"],
  " objects, including plots.\n\nYou've learned that ",
  StyleBox["you should always clear variables before using them, ",
    FontWeight->"Bold"],
  "so you won't have any nasty surprises.\n\nYou've learned how to ",
  StyleBox["solve polynomial equations ",
    FontWeight->"Bold"],
  "with the ",
  StyleBox["Solve",
    FontFamily->"Courier"],
  " command, if you want the roots exactly; or how to find the roots \
numerically with ",
  StyleBox["NRoot",
    FontFamily->"Courier"],
  ", if a numerical approximation is sufficient.  You can even solve very \
difficult transcendental equations (\"transcendental\" just means \"more \
complicated than algebraic\") if you can make a close-enough guess at a 
root.\
\n\nYou've learned that ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " understands trig functions, but you have to be very patient to get the \
answers you expect.\n\nFinally, you've learned how to construct lists of \
numbers using the ",
  StyleBox["Table",
    FontFamily->"Courier"],
  " command, and how to plot a list (using ",
  StyleBox["ListPlot",
    FontFamily->"Courier"],
  " instead of ",
  StyleBox["Plot",
    FontFamily->"Courier"],
  ")."
}], "Text",
  Evaluatable->False,
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  "Two of the roots of the cubic equation we solved were complex numbers, not 
\
real numbers.  That means one root was real.  Do you notice anything about \
the complex roots?  (It might be easier to look at the numerical values than 
\
the exact values.)\n\nTry solving a fourth-degree equation, and look at its \
complex roots.  Finally, use ",
  StyleBox["NRoots",
    FontFamily->"Courier"],
  " to solve a few 5th-degree polynomial equations.\n\nFormulate conjectures 
\
(a) about the number of roots an equation has; (b) something special about \
the complex roots; and (c) about whether cubic equations must always have a \
REAL root."
}], "Text",
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  "A ",
  StyleBox["rational number ",
    FontWeight->"Bold"],
  "is one which is the quotient of two integers.  More than two thousand \
years ago the Greeks discovered that the square root of 2 is ",
  StyleBox["not ",
    FontWeight->"Bold"],
  "a rational number.  We believe (we're not sure) they reasoned as follows: 
\
if Sqrt[2] = m/n, where m and n are integers and the fraction is reduced to \
lowest terms, then m^2 = 2n^2.  Since the right-hand side of this equation is 
\
an even integer, so must the left-hand side be an even integer.  But if m^2 \
is even, then m must be even (because the square of an odd integer is odd). 
\
Therefore m is of the form 2r, where r is an integer, and therefore m^2 = \
2n^2 becomes 4r^2 = 2n^2, i.e. n^2 = 2r^2.  Now by exactly the same \
reasoning, n^2 is even, hence n is even.  But now m and n are both even, \
which contradicts our assumption that the fraction was reduced to lowest \
terms!  The contradiction means that our supposition that sqrt(2) is rational 
\
is untenable, i.e. sqrt(2) is not rational.\n\nIt has been said that any \
person claiming to be educated should know the proof that sqrt(2) is \
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irrational; your education has now begun in earnest.\n\nBut here's one which \
is a little more difficult:  we calculated the solution of x^x = 2 to many \
decimal places, and didn't see any obvious patterns.  Prove that, in fact, \
the solution is not a rational number."
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  "In the notebook we looked at solutions of the equation x cos(x) = sin(x). \
Dividing both sides by cos(x), this is the same as x = sin(x)/cos(x), i.e. x \
= tan(x).  (We can't have cos(x) = 0, because that would imply sin(x) = 0, \
and the sine and cosine of an angle can't ",
  StyleBox["both ",
    FontSlant->"Italic"],
  "be 0.)\n\nTry solving x = tan(x) for the root near 7 by using ",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  ", and also graphically, by plotting y = x and y = tan(x) on the same \
graph."
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  StyleBox["'s Choices",
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  "There are infinitely many solutions of x cos(x) = sin(x) (remember what \
simultaneously graphing x and tan(x) showed).  Which one does",
```

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  StyleBox[" FindRoot ",
    FontFamily->"Courier"],
  "pick?\n\nDifferent starting points will yield different roots.  We'll get 
\
you started by giving you a function which returns the found root as a \
function of the starting guess:"
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    \(f[s_] := 
      x /. FindRoot[x\ Cos[x] == Sin[x], {x, s}, MaxIterations \[Rule] 
50]\)], 
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  "There are several new features in this construct:  first, the very idea \
that we're defining a ",
  StyleBox["function.  ",
    FontSlant->"Italic"],
  " Second, the ",
  StyleBox["argument ",
    FontSlant->"Italic"],
  "of the function, what we plug into it, is s; on the left-hand side of the 
\
definition it's followed by an underscore to warn ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " that ",
  StyleBox["this is the argument of the function. ",
    FontSlant->"Italic"],
  StyleBox[" ",
    FontWeight->"Bold"],
  "On the right-hand side, s appears without the underscore. \n\nSecond, the 
\
\"equals\" sign is written :=.  This is \"deferred assignment\"; we don't \
want ",
  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " to calculate anything yet, we're just defining the function f, and := is 
\
the way to do this.\n\nThird, we have the \"given that\",  /. construction.  
\
The right-hand side is read \"the value of x, GIVEN the results of ",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  ".\"  Since the output of ",
  StyleBox["FindRoot",
    FontFamily->"Courier"],
  " is a rule, x->\"something\", we see that the value of x, given that x \
becomes \"something\", is \"something\".  Roundabout, but it works.\n\n\
Finally, we've upped the maximum iterations that",
  StyleBox[" FindRoot",
    FontFamily->"Courier"],
  " will do to 50.  This still may not be enough (don't worry, ",
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  StyleBox["Mathematica",
    FontSlant->"Italic"],
  " will complain if it's not enough).\n\nNow try plotting y = f(x) for \
various ranges of x; e.g., 6 \[LessEqual] x \[LessEqual] 8.  Remember, the \
y-value is always a solution of ",
  StyleBox["y Cos[y] = Sin[y]",
    FontFamily->"Courier"],
  ".  How regular is this graph?"
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  "The function ",
  StyleBox["PrimePi[x] ",
    FontFamily->"Courier"],
  "returns the ",
  StyleBox["number of primes less than or equal to x.  ",
    FontSlant->"Italic"],
  "For example, ",
  StyleBox["PrimePi[7]",
    FontFamily->"Courier"],
  " is 4, because {2, 3, 5, 7} is the list of prime numbers less than or \
equal to 7, and there are 4 of them.\n\nA famous theorem of mathematics says 
\
that the quantity",
  StyleBox[" \nPrimePi[x] / (x/Log[x])",
    FontFamily->"Courier"],
  " \napproaches 1 as x approaches infinity.  (This is very difficult to \
prove.)  Roughly speaking, that means that",
  StyleBox[" PrimePi[x]",
    FontFamily->"Courier"],
  " and",
  StyleBox[" x/Log[x]",
    FontFamily->"Courier"],
  " are approximately equal, but only in a percentage sense; the difference \
between them may be quite large, but that difference is small ",
  StyleBox["compared to the quantities",
    FontSlant->"Italic"],
  ".\n\nCheck this out by ",
  StyleBox["ListPlotting",
    FontFamily->"Courier",
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  " the difference",
  StyleBox[" ",
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  StyleBox["PrimePi[x]-x/Log[x]",
    FontFamily->"Courier",
    FontSize->12],
  " for, say, 2 \[LessEqual] x \[LessEqual] 1000.  Now ",
  StyleBox["ListPlot",
    FontFamily->"Courier",
    FontSize->12],
  " the RATIO",
  StyleBox[" ",
    FontFamily->"Courier"],
  StyleBox["PrimePi[x]/(x/Log[x])",
    FontFamily->"Courier",
    FontSize->12],
  " in the same range.  Would you guess from the graph of the ratio that the \
\
limit is 1?"
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  "Try making plots of the binomial coefficients ",
  StyleBox["Binomial[n,k]",
    FontFamily->"Courier",
    FontSize->12],
  " for 0 \[LessEqual] k \[LessEqual] n for different values of n.  You folks \
\
took the SAT exams not so long ago; does this curve look familiar?"
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