The University of Southern California

MATH 226 Fall 2016

Final Exam

Last Name: ______________________________

First Name: ______________________________

Signature: ______________________________

Please indicate your instructor and lecture time below:

G. Reyes Souto	F. Malikov	N. Emerson	N. Haydn	S. Lototsky	A. Soibelman
9 am	10 am	11 am	12 pm	12 pm	1 pm
10 am	11 am				

Instructions: Please show all of your work and reasoning. You may use one 8.5-by-11-inch formula sheet, written in your own handwriting on both sides. No other notes, books, calculators, electronic devices, or other memory aids are allowed for use during the test. All electronic devices, including mobile phones, must be turned off. Numerical answers should be simplified. If you need more space than what is provided, please use the back of the previous page, indicating clearly where the solution is continued. The exam lasts two hours (120 minutes).

Problem	Possible	Actual	Problem	Possible	Actual	Problem	Possible	Actual
1	20		4	30		7	25	
2	30		5	10		8	20	
3	20		6	20		9	25	
	70			**60**			**70**	
Total								

1. Consider two planes, $\mathcal{P}_1 : x + y = 27$ and $\mathcal{P}_2 : 2x + z = 10$, and a point $Q = (3, 4, 1)$.

(a) Write an equation of the plane that passes through the point Q and is perpendicular to the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.
`Key words: cross product.`

(b) Write a parametric equation of the line that passes through the point Q and is parallel to the planes $\mathcal{P}_1$ and $\mathcal{P}_2$.

2. The height of a hill above the xy-plane is $z = 10 - 2x^2 - y^2$.

(a) At the point $(1, 1, 7)$ on the hill, what is the maximum possible rate of change of height?

(b) An ant moves on the hill in such a way that the coordinates of the ant on the xy-plane at time t are

$$x(t) = t, \quad y(t) = \sqrt{t}.$$

[i] At the point $(1, 1)$ on the xy-plane, what is the angle between the direction in which the ant is moving and the direction in which the hight of the hill increases most rapidly?

[ii] At the time $t = 1$, what is the rate of change of ant's height?

3. The dimensions of a **closed rectangular box** are found by measurement to be 10cm by 15cm by 20cm, but there is the possibility of error ± 0.1cm in each measurement. Use linear approximation to estimate the range of possible values for the total surface area of the box.

4. Consider the function $f(x, y) = x^4 + y^4 - 4xy$.

(a) Identify and classify all critical points of the function f.
`Key words:  Second partials test.`

(b) Determine the minimum and maximum values of the function f on the **curve**

$$x^4 + y^4 = 32.$$

`Key words: Lagrange multipliers.`

(c) Determine the absolute minimum and maximum values of the function f on the **region**

$$x^4 + y^4 \leq 32.$$

5. Compute the iterated integral

$$\int_0^9 \int_{\sqrt{y}}^3 e^{x^3} dx\ dy$$

by reversing the order.

6. The planar region G consists of two semi-disks $0 \le y \le \sqrt{1-x^2}$ and $-\sqrt{4-x^2} \le y \le 0$. Consider the vector field

$$\boldsymbol{F}(x,y) = (x^2+y^2)^{3/2}\big(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}}\big).$$

Compute the line integral of the vector field $\boldsymbol{F}$ along the boundary of G oriented counter-clockwise.

`Key words: Green's theorem, polar coordinates.`

7. Let G be the region $x \geq 0,\ y \geq 0,\ z \geq 0,\ x^2 + y^2 + z^2 \leq 4$. Compute the flux of the vector field

$$\boldsymbol{F}(x, y, z) = \langle xz^2 + y^3,\ \sin(x + z) + z^3,\ e^{xy} \rangle$$

out of G.

`Key words:  Divergence theorem, spherical coordinates.`

8. (a) Consider the vector field $\boldsymbol{F}(x, y, z) = \langle 2x - ay,\ 3y^2 + 3x,\ 1\rangle$. Determine the value of a for which $\boldsymbol{F}$ is conservative and compute a potential function corresponding to this value of a.

(b) Evaluate the line integral

$$\int_C \boldsymbol{G} \cdot d\boldsymbol{r},$$

where $\boldsymbol{G}(x, y, z) = \nabla g(x, y, z)$, $g(x, y, z) = x^2 + 3xy + y^3 + z$, and C is the curve defined by $\boldsymbol{r}(t) = \langle 1 + 2\cos t,\ -3\sin t,\ t\rangle,\ 0 \le t \le \pi$.

9. Evaluate the line integral $\oint_C \left(\sin(1+x^3)+3y\right) dx - \left(2z+e^{y^2}\right) dy + \left(5y-\sqrt{1+z^4}\right) dz$, where C is the ellipse $x^2+y^2=1,\ 3x+4y+z=12$ oriented counterclockwise as seen from above.

`Key words: Stokes' theorem, curl.`