MATH 226 **FINAL EXAM** **December 6, 2017**

Last Name: ______________________ **First Name:** ______________________

Student ID Number: ____________________ **Signature:** ____________________

Circle the name of your instructor and your lecture time:

Baxendale (10AM) Bonahon (11AM) Bonahon (1PM) Hall (9AM)

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INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed. You may use your own formula sheet. The exam is worth a total of 200 points.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
Total	200	

Problem 1.

a. Find an equation for the plane passing through the points $(2, 3, 1)$, $(1, 6, 4)$ and $(3, 1, 0)$.

b. Let C be the curve parametrized by $\mathbf{r}(t) = \langle t^2, t+3, 2t^2+t \rangle$. Find parametric equations for the line which is tangent to this curve C at the point $(4, 1, 6)$.

c. Find the point where the tangent line in Part **b** meets the plane in Part **a**.

Problem 2. Let S be the surface of equation $xz^3 + x^2y - y^2z = 5$.

a. Find an equation of the tangent plane of S at the point $P(2, 3, 1)$.

b. Suppose that $Q(2.01, 2.96, z)$ is a point on the surface S near to P. Use calculus to estimate the value of z.

Problem 3. Find all critical points (x, y) of the function $f(x, y) = \frac{1}{4}x^4 - x^3 + 2xy - y^2$, and classify each one as a local maximum, a local minimum or neither.

Problem 4. [This problem extends over two pages.]

a. Find the absolute maximum and minimum values of the function

$$f(x, y) = 3x^2 + 2y^2 + 2y - 3$$

over the circle of equation $x^2 + y^2 = 4$.

b. Find the absolute maximum and minimum values of the same function

$$f(x, y) = 3x^2 + 2y^2 + 2y - 3$$

over the closed disk consisting of those points (x, y) such that $x^2 + y^2 \leqslant 4$.

Problem 5.

a. Let E be the region consisting of those points (x, y, z) in the first octant (where $x \geq 0$, $y \geq 0$, $z \geq 0$) and under the paraboloid of equation $z = 2 - x^2 - 2y^2$. Find the expressions to replace all the asterisks in the iterated integral

$$\iiint_E f(x, y, z)\, dV = \int_*^* \int_*^* \int_*^* f(x, y, z)\, dy\, dx\, dz.$$

Note the order of integration. Do not attempt to compute the integral!

b. Change the order of integration and evaluate

$$\int_0^4 \int_{\sqrt{y}}^2 \cos(x^3)\, dx\, dy.$$

Problem 6.

Consider the curve C parametrized by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ for $0 \leqslant t \leqslant 1$.

a. Calculate the line integral $\displaystyle\int_C f\, ds$ of the scalar function $f(x, y, z) = 2x + 9z$.

b. Calculate the line integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ of the vector field $\mathbf{F}(x, y, z) = y\mathbf{i} + xz\mathbf{j} + \mathbf{k}$.

Problem 7. For the vector field

$$\mathbf{F}(x, y, z) = \langle 2xyz^3 + y, x + x^2z^3 - z^2, 3x^2yz^2 - 2yz \rangle,$$

show that the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ depends only on the endpoints of the curve C, and evaluate it when C is any path going from $(0, 0, 0)$ to $(3, 1, 2)$.

Problem 8. Evaluate the line integral

$$\int_C (xy^4 + xy)\,dx + (2x^2y^3 + xy)\,dy$$

where C is the closed piecewise linear path from $(1, 1)$ to $(4, 4)$ to $(1, 4)$ and back to $(1, 1)$. You may find it convenient to use one of the theorems discussed in the course.

Problem 9. Let the surface S be the portion of the plane of equation $x + y - z = 0$ where $x^2 + y^2 \leqslant 4$, $x \geqslant 0$ and $y \geqslant 0$. Compute the surface integral $\displaystyle\iint_S (x^2 - y^2 + z^2)\, dS$.

Problem 10. Let E be the solid consisting of those points that sit above the cone of equation $z = 2\sqrt{x^2 + y^2}$ and inside of the sphere of equation $x^2 + y^2 + z^2 = a^2$. Calculate the flux

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

of the vector field $\mathbf{F}(x, y, z) = \langle x^3 z + 2xy, xz^2 - y^2, x^2 z^2 \rangle$ outwards across the boundary S of E. You may find it convenient to use one of the theorems discussed in the course.