Math 125 Calculus I — Spring 2015

Final Exam — May 9th, 2015

Name:

USC ID :

Signature:

Please circle your lecture time:

9am Sobaje	10am Sobaje	11am Reyes Souto	12pm Kamienny	1pm Reyes Souto	2pm Yıldırım

- The duration of the exam is 120 minutes.
- In order to receive full credit, please show all of your work and justify your answers.
- This is a closed book, closed notes exam. No calculators or other electronic devices.
- If you need more space, you can write on the back of a page as long as you clearly indicate this in your solution. More paper is also available at the front of the room.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Total
/30	/20	/20	/20	/20	/20	/20	/20	/30	/200

1. (30 points) Evaluate the following limits. You may not use l'Hôpital's rule. Your answer should be written in the simplest possible form.

   (a) (6 points) $\lim_{x \to 0} \dfrac{\tan x}{x^2 - x}$

   (b) (6 points) $\lim_{x \to \infty} \left(x - \sqrt{x^2 + x + 1} \right)$

(c) (6 points) $\lim_{x \to \infty} x^{-4} \cos(x^4)$

(d) (6 points) $\lim_{x \to 3^-} \frac{x^2 - 9}{|x - 3|}$

(e) (6 points) $\lim_{x \to 4} \frac{\ln x - \ln 4}{x - 4}$

2. (20 points) Compute the derivatives of the following functions. You do not need to simplify your answer.

   (a) (5 points) $f(x) = \dfrac{e^x}{x^3 - 1}$

   (b) (5 points) $f(x) = x^{\ln x}$

(c) (5 points) $f(x) = \ln[(x^3 + 3)^3(x^2 + 2)^2(x + 1)]$

(d) (5 points) $f(x) = \int_{2x}^{x^2} \sin t \, dt$

3. (20 points) Evaluate the following integrals. You do not need to simplify your answer.

(a) (5 points) $\displaystyle\int \frac{x^2 + 3x + 4}{\sqrt{x}}\, dx$

(b) (5 points) $\displaystyle\int_1^e \frac{\ln x}{x}\, dx$

(c) (5 points) $\int \frac{\sin \sqrt{x}}{\sqrt{x}}\, dx$

(d) (5 points) Find the area lying between the graph of $y = x^3 + 1$, and the x-axis over the interval $[-1, 4]$.

4. (20 points) Consider the curve given by the equation $y = \ln(x^3 + y^3)$. Find the equation of the tangent line at $(1, 0)$.

5. (10 points) Find the linear approximation of the function $g(x) = \sqrt[3]{1+x}$ at $a = 0$, and use it the approximate the number $\sqrt[3]{0.95}$.

6. (a) Formulate the Mean Value Theorem.

(b) What is the maximum number of the solutions of the equation $g(x) = 0$ if the equation $g'(x) = 0$ has exactly two real solutions.

(c) If $f(1) = 2$ and $|f'(x)| \leq 4$ for $1 \leq x \leq 3$, how small and large can $f(x)$ possibly be on $[1, 3]$?

7. (20 points) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

8. (20 points) The half-life of Ca-14 is 5730 years.

   (a) (8 points) Find a formula for the amount of Ca-14 present after t years in a sample that initially contains A grams.

   (b) (4 points) How long will it take for the sample to degrade so that it contains only one third of its initial amount?

   (c) (8 points) If the initial sample contains 100 grams of Ca-14 how long will it take until there are only 10 grams left?

9. Consider the function $f(x)$ given below. The first and second derivatives of f are also provided (you do not need to check them):

$$f(x) = \frac{x}{x^3 - 1}, \qquad f'(x) = -\frac{2x^3 + 1}{(x^3 - 1)^2}, \qquad f''(x) = \frac{6x^2(x^3 + 2)}{(x^3 - 1)^3}.$$

(a) Find the domain of $f(x)$.

(b) Find all asymptotes of $f(x)$. Each vertical asymptote you find should be described by two one-sided limits, and each horizontal asymptote by one limit.

(c) Find the intervals where $f(x)$ is increasing or decreasing.

(d) Find the x and y coordinates of all critical points of $f(x)$ and indicate whether each is a local maximum, a local minimum, or neither. If there are no critical points, say so.

(e) Find the intervals where f is concave-up or concave-down.

(f) Find the x and y coordinates of all points of inflection of $f(x)$. If there are no points of inflection, say so.

(g) Sketch the graph of $f(x)$. Your graph should clearly reflect all the information you found in the previous parts.