Math 125 Final Exam
December 7th, 2016

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam! Also, check the box next to the class for which you are registered.

You must **show all of your work and justify your methods** to obtain full credit. Circle your final answers. Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed, but you may use the double sided HANDWRITTEN sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Name (please print): ______________________________

Signature: ______________________________

Student ID: ______________________________

- ☐ L. Wolcott 9 AM
- ☐ L. Wong 10 AM
- ☐ J. Toulisse 12 PM
- ☐ N. Tiruviluamala 1 PM
- ☐ D. Searles 9 AM
- ☐ N. Tiruviluamala 11 AM
- ☐ D. Searles 12 PM
- ☐ G. Reyes Souto 2 PM
- ☐ D. Crombecque 10 AM
- ☐ A. Asok 11 AM
- ☐ D. Crombecque 1 PM

Do not write on this page below this line.

1 (20 pts)	6 (20 pts)
2 (25 pts)	7 (15 pts)
3 (20 pts)	8 (25 pts)
4 (15 pts)	9 (15 pts)
5 (25 pts)	10 (20 pts)

200 Points Total

1. (20 points) Evaluate the following limits and justify your responses. Do **not** use L'Hôpital's rule.

   (a) $\displaystyle\lim_{x\to -2} \frac{x^2-4}{x^2+x-2}$

   (b) $\displaystyle\lim_{t\to 0} \frac{\sqrt{t^2+16}-4}{t^2}$

   (c) $\displaystyle\lim_{x\to 2} \frac{x^x-4}{x-2}$

   Hint: Interpret this limit as a derivative.

2. (25 points) $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) + 3x & \text{for} \quad x \neq 0 \\ 0 & \text{for} \quad x = 0 \end{cases}$

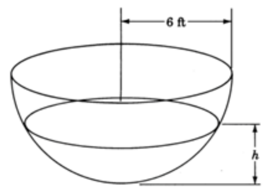
(a) Is f differentiable at 0? Justify.
If it is, write the equation of the tangent line to the graph of f at the point where $x = 0$.

(b) Compute $f'(x)$ for all $x \neq 0$

(c) Is f continuous at 0? Justify.

(d) Use a method that you learned in this course to approximate $f(0.1)$.

3. (20 points) A hemispherical tank of radius 6 feet



is being filled with water. At the instant when the height h of the water is 2 feet, the water level is rising at a rate of 1 foot per hour. Find the rate at which the area of the top circular surface of the water is increasing at this instant. (You may consult the last page of this exam packet for any formulas that you might need.)

4. (15 points) Consider the curve given by the equation

$$e^{xy}\ln(y) + e^2 = y^2 + 1.$$

Find the slope of the tangent line to this curve at the point $(0, e)$.

5. (25 points) Consider the function:

$$f(x) = \frac{x}{x^2 - 1}$$

The first and second derivatives of this function are given by the formulas:

$$f'(x) = \frac{-x^2 - 1}{(x^2 - 1)^2} \qquad\qquad f''(x) = \frac{2x^3 + 6x}{(x^2 - 1)^3}$$

In all of the following questions, justify your answers in an organized manner.

(a) What are the horizontal and vertical asymptotes of f (if any)?

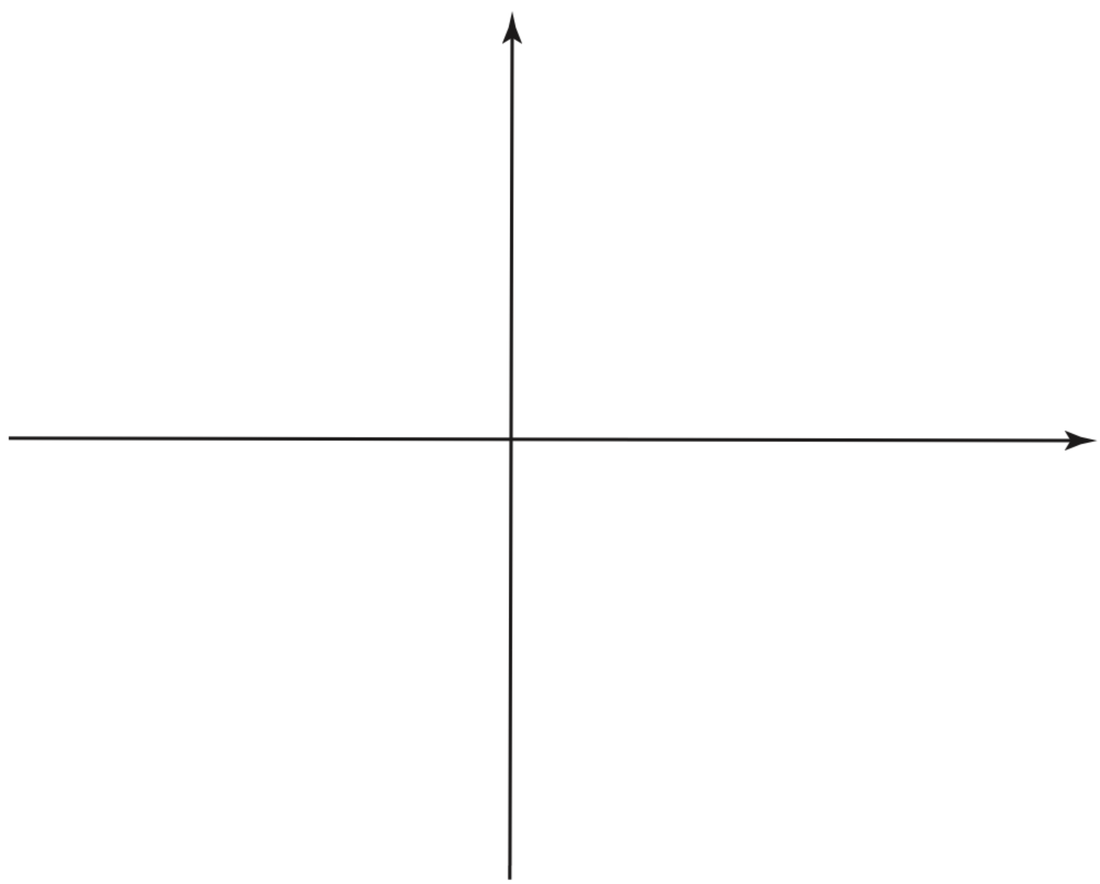
(b) On which interval(s) is f increasing?

(c) What are the x-values of the local maxima/minima of f (if any)?

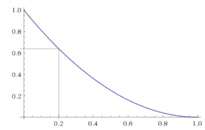
(d) On which interval(s) is f concave down?

(e) What are the x-values of the points of inflection of f (if any)?

(f) Sketch a graph of f below and label the x-intercepts, asymptotes, relative extrema, and points of inflection.



6. (20 points) A rectangle is to be inscribed in the region enclosed by the $x-$axis, the $y-$axis, and the curve $y = (x-1)^2$.



(a) What is the maximum area that such a rectangle can have? Justify your answer.

(b) What is the maximum perimeter that such a rectangle can have? Justify your answer.

7. (15 points) Assume that the acceleration (in meters per second squared) of a particle at time t (in seconds) is given by

$$a(t) = \sin\left(\frac{t}{2}\right).$$

The particle is at position $x = 1$ meters at time $t = 0$ seconds and it is at position $x = 8$ meters at time $t = \pi$ seconds. Find the position $x(t)$ of the particle as a function of time. (Recall that the acceleration of a particle is the derivative of its velocity.)

8. (25 points) Evaluate the following integrals:

(a) $\displaystyle\int 10^x \cos 10^x \, dx$

(b) $\displaystyle\int \frac{x + x^2 + x^5}{x^{5/2}} \, dx$

(c) $\int x^7\sqrt{x^4+3}dx$

(d) $\int_0^{\sqrt{3}} x\sqrt{9-x^4}dx$

Hint: You should make the substitution $u = x^2$ and then use geometry.

9. (15 points)Let $g(x)$ be defined as

$$g(x) = \int_{x^5+1}^{2} t(t+2)^{\frac{1}{2}}\,dt.$$

(a) Find $g'(x)$.

(b) Find $\left[g^{-1}\right]'(0)$.

10. (20 points) The temperature of a glass is measured to be 27° C at 9 pm and 26° C an hour later. The glass is in an air-conditioned room where the temperature is held constant at 20° C. The glass was removed from the dishwasher at a temperature of 37° C. You may use the fact that the temperature T (in degrees Celsius) will obey the equation

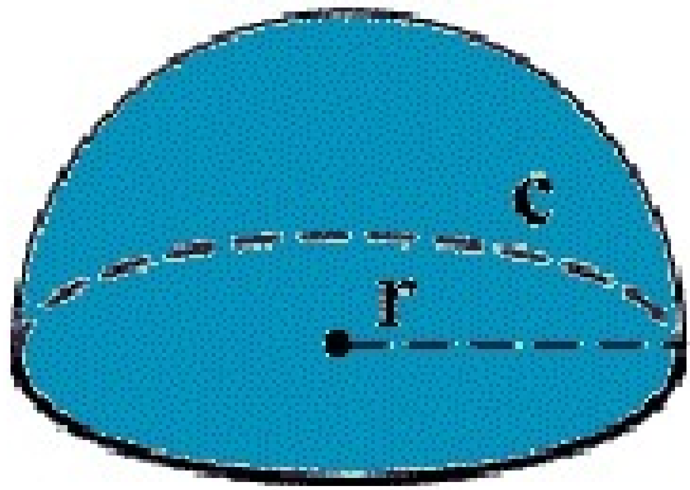
$$T(t) = A + Be^{-kt},$$

where t represents the number of hours after 9 pm.

(a) Using the fact that in the long run, the temperature of the glass approaches the temperature of the room, what is $\lim_{t \to \infty} T(t)$? Use this and the information given to solve for all the constants A, B, and k.

(b) When was the glass removed from the dishwasher?

Hemisphere & *Formulas*



$$surface\ area = 2\pi\, r^2$$

$$volume = \frac{2\pi\, r^3}{3}$$