Math 125 • Calculus I

Final Exam, December 9, 2015

Name:_____

ID:_____

Circle your professor and class time if applicable:

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Soibelman (11a, 1p), Tokorcheck (9a, 12p), Reyes (12p, 2p)

INSTRUCTIONS

Answer all the questions. You must show your work to obtain full credit. Points may be deducted if you do not justify your final answer. Please indicate clearly whenever you continue your work on the back of the page. Calculators are not allowed.

Question	Points Earned	Points Possible
Q 1		25
Q 2		20
Q 3		15
Q 4		10
Q 5		20
Q 6		25
Q 7		25
Q 8		20
Q 9		20
Q 10		20
Total:		200

1. Using the procedures from this course (e.g. not L'Hospital's rule), find each limit or explain why it does not exist.

(a)
$$\lim_{x \to 3} \frac{2x^2 - 5x - 3}{x^2 + 4x - 21}$$

(b)
$$\lim_{x \to 0} \frac{\sin^2(5x)}{x \sin(2x)}$$

(c)
$$\lim_{x \to \infty} \sqrt{x^4 + x^2 + 5} - x^2$$

(d)
$$\lim_{x \to 0} x^4 \sin\left(\frac{1}{x}\right)$$

2. Let $f(x) = \begin{cases} x^2, & x \leq 1 \\ a + b\left(x - \frac{1}{x}\right), & x > 1 \end{cases}$

(a) For what values of a and b is f continuous everywhere?

(b) For what values of a and b is f differentiable everywhere?

3. Find the derivatives of:

(a) $f(x) = \ln(11x^2 - 8x)$

(b) $f(x) = x^5 \, 5^x$

(c) $f(x) = \displaystyle\int_5^{x^3} \frac{\sec(7t^3)}{\sqrt{6t^5 + 91t^2}} \, dt$

4. Find the equation of the line tangent to the given curve at the point $(1, 0)$:

$$y + \cos(xy^2) + 3x^2 = 4.$$

5. Show that the function $f(x) = x^3 + 2x - \sin(x)$ has exactly one zero.

6. Given the function $f(x) = \frac{1}{12}(x-2)^2(x+7)$ and its two derivatives

$$f'(x) = \frac{1}{4}(x-2)(x+4), \quad f''(x) = \frac{x}{2} + \frac{1}{2}$$

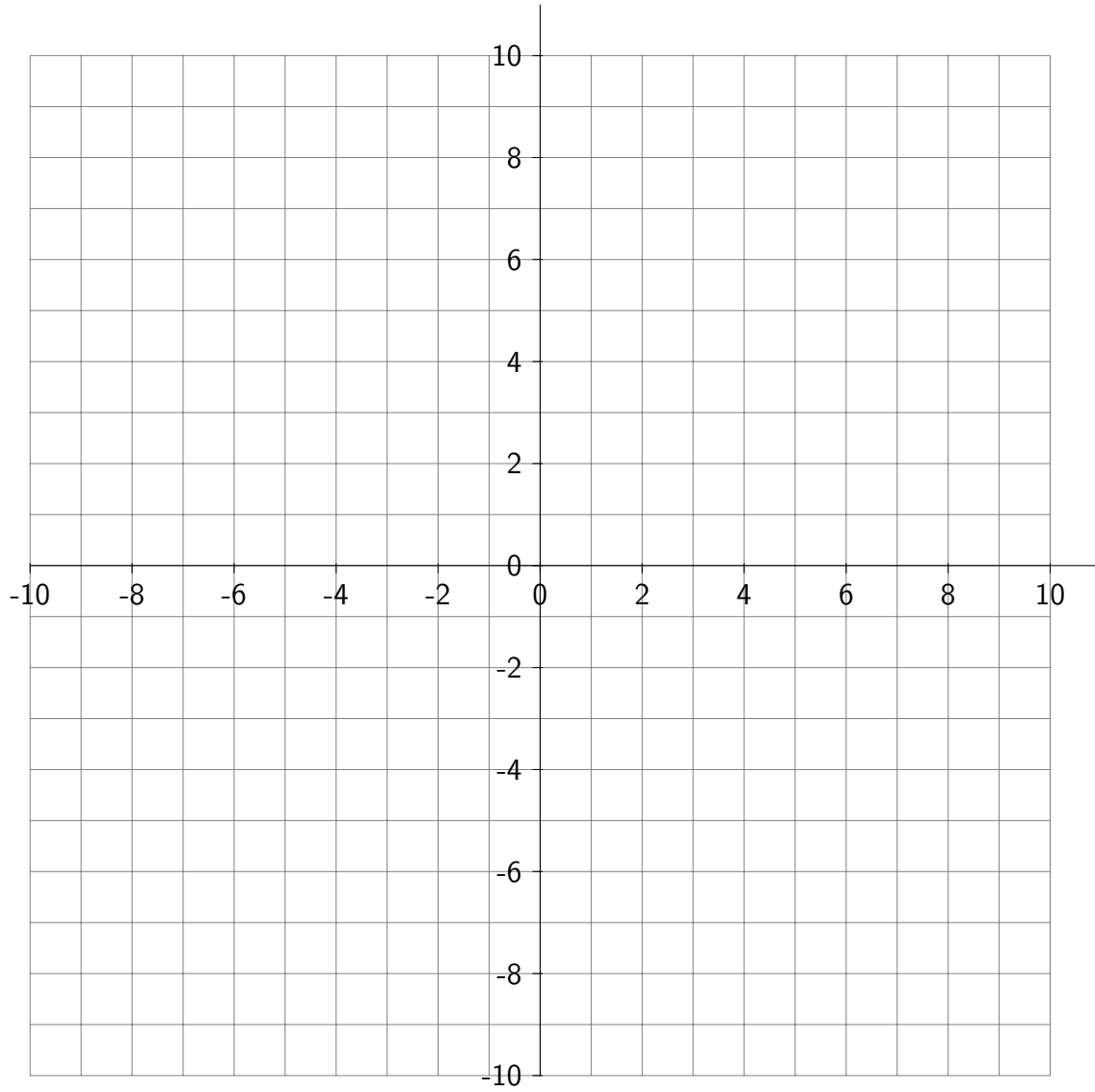(a) Find the x and y intercepts.

(b) Find the intervals of increase/decrease.

(c) Find the intervals of concavity.

(d) Locate all local maxima and minima (if there are none, say so).

(e) Locate all inflection points (if there are none, say so).

(f) Sketch the graph on the axes provided.

7. Find

(a) $\displaystyle\int \left(9t^{-2/3} + 4t^{7/3}\right) \, dt$

(b) $\displaystyle\int_1^e \frac{(\ln x)^2}{x} \, dx$

(c) $\int t^2 \sec^2(9t^3 + 1) \, dt$

(d) $\int_0^5 15x\sqrt{x + 4} \, dx$

8. A tank of water in the shape of a cone is leaking water at a constant rate of $1 \text{ ft}^3/\text{min}$. The base radius of the tank is 5 ft and the height of the tank is 14 ft.

(a) At what rate is the depth of the water in the tank changing when the depth of the water is 6 ft?

(b) At what rate is the radius of the top of the water in the tank changing when the depth of the water is 6 ft?

9. The number of bacteria in a culture grows at a rate proportional to the total number of bacteria. The starting population contains 200 cells. After four hours there are 1000 cells.

(a) Find the expression for the number of bacteria after t hours.

(b) How much time will pass before the population reaches 5000 cells?

10. 1200 cm^2 of material is available to make a cylindrical barrel (a container with a circular base). If the top of the barrel is open, find the largest volume of the barrel.