Problem 1. (12 points) The following chart records estimates for the dew point D (in degrees celsius) for various values of the room temperature T and the relative humidity H.

DEW POINT INDEX															
		Relative Humidity (%)													
		30	35	40	45	50	55	60	65	70	75	80	85	90	95
Room Temperature (°C)	30	10.5	12.9	14.9	16.8	18.4	20.0	21.4	22.7	23.9	25.1	26.2	27.2	28.2	29.1
	29	9.7	12.0	14.0	15.9	17.5	19.0	20.4	21.7	23.0	24.1	25.2	26.2	27.2	28.1
	28	8.8	11.1	13.1	15.0	16.6	18.1	18.1	19.5	20.8	22.0	23.2	24.2	25.2	26.2
	27	8.0	10.2	12.2	14.1	15.7	17.2	18.6	19.9	21.1	22.2	23.3	24.3	25.2	26.1
	26	7.1	9.4	11.4	13.2	14.8	16.3	17.6	18.9	20.1	21.2	22.3	23.3	24.2	25.1
	25	6.2	8.5	10.5	12.2	13.9	15.3	16.7	18.0	19.1	20.3	21.3	22.3	23.2	24.1
	24	5.4	7.6	9.6	11.3	12.9	14.4	15.8	17.0	18.2	19.3	20.3	21.3	22.3	23.2
	23	4.5	6.7	8.7	10.4	12.0	13.5	14.8	16.1	17.2	18.3	19.4	20.3	21.3	22.2
	22	3.6	5.9	7.8	9.5	11.1	12.5	13.9	15.1	16.3	17.4	18.4	19.4	20.3	21.2
	21	2.8	5.0	6.9	8.6	10.2	11.6	12.9	14.2	15.3	16.4	17.4	18.4	19.3	20.2
	20	1.9	4.1	6.0	7.7	9.3	10.7	12.0	13.2	14.4	15.4	16.5	17.4	18.3	19.2
	19	1.0	3.2	5.1	6.8	8.3	9.8	11.1	12.3	13.4	14.5	15.5	16.4	17.3	18.2
	18	0.2	2.3	4.2	5.9	7.4	8.8	10.1	11.3	12.5	13.5	14.5	15.4	16.3	17.2
	17	-0.6	1.4	3.3	5.0	6.5	7.9	9.2	10.4	11.5	12.5	13.5	14.5	15.4	16.2
	16	-1.4	0.3	2.4	4.1	5.6	7.0	8.3	9.4	10.5	11.6	12.6	13.5	14.4	15.2

(Chart from https://www.alldamp.co.uk/condensation)

(a) (6 points) Using the table above, estimate the values of the partial derivatives $\frac{\partial D}{\partial T}$ and $\frac{\partial D}{\partial H}$ when $T = 20$ degrees and $H = 60$. You must clearly indicate how you perform the estimation.

(b) (6 points) Using the table and your answer to part (a), estimate the dew point for a room temperature of $T = 19.7$ degrees and relative humidity of $H = 63$ percent.

Problem 2. (18 points) You are interested in building a new house on a vacant lot in Altadena. Due to the size of the lot, the house will have to be between 600 and 2000 square feet in size. You know the marginal cost of building an x square foot house is:

$$MC(x) = 500 - \frac{x}{10} \quad \text{dollars/square foot}$$

(a) (4 points) If the fixed costs of building are \$100,000, find the cost function $C(x)$.

(b) (3 points) Find the average cost function $AC(x)$, and show by direct calculation that $AC(x) > MC(x)$ for all $x > 0$.

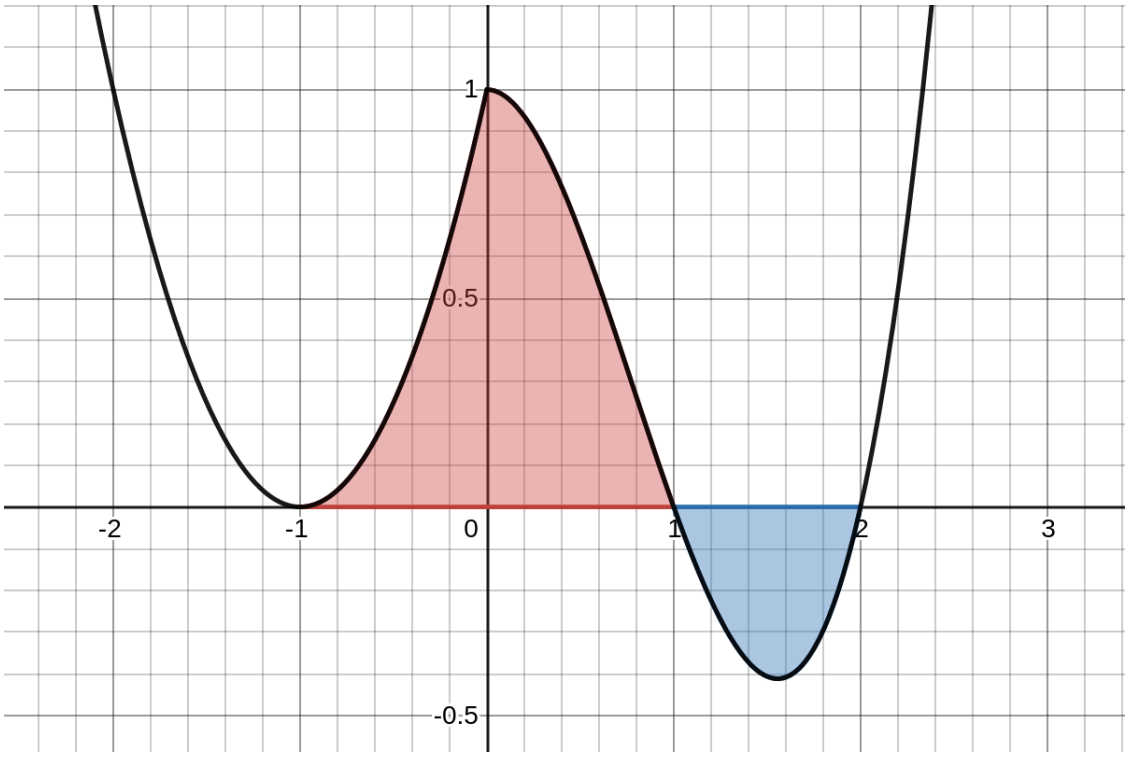
(c) (3 points) How does $AC(x)$ differ in meaning from $MC(x)$? Explain in the context of the problem.

(d) (4 points) Find the minimum average cost per square foot for construction, given the limitations on the lot size. Be sure to show all calculations, and justify that your answer is indeed the minimum.

(e) (4 points) Determine when the difference between $AC(x)$ and $MC(x)$ is as small as possible. Be sure to show all calculations, and justify that your answer is indeed the minimum.

Problem 3. (18 points) For some function $f(x)$, the graph of its derivative $f'(x)$ is shown below. You may assume the region shaded red has larger area than the region shaded blue.

Graph of the Derivative $f'(x)$



(a) (3 points) How many critical points does $f(x)$ have on the interval shown? Explain how you identify the critical points, and give approximate x-values for each.

(b) (3 points) How many inflection points does $f(x)$ have on the interval shown? Explain how you identify the inflection points, and give approximate x-values for each.

(c) (6 points) For each of the critical points in part (a), say whether it is a local maximum, local minimum, or neither, justifying your conclusion with an appropriate test.

(d) (6 points) State the x-values where $f(x)$ achieves its global maximum and minimum on the interval $[-1, 2]$. Justify your answer.

Problem 4. (12 points) You sell a software product that you developed and receive three payment options:

- Option I is to receive a lump sum payment of \$1,000,000 now.
- Option II is to receive a lump sum payment of \$1,750,000 ten years from now.
- Option III is to receive payment at a rate of $5000t^2$ dollars/year for the next ten years, where t is time from the present in years.

Assume you can get 5% interest, compounded continuously.

Which option leaves you with the most money after ten years? Be sure to justify your answer, and show all work in your calculations in order to receive full credit.

Problem 5. (18 points) From *Wikipedia*:

> In computer architecture, *Amdahl's law* is a formula which gives the theoretical speedup in latency of the execution of a task at fixed workload that can be expected of a system whose resources are improved.

The formula given is $S = \dfrac{1}{(1-x) + \frac{x}{y}}$, where

- S is the theoretical speedup of the execution of the whole task
- x is the proportion of the original task that benefits from improvements
- y is the speedup of that part of the task that benefits from improvements

Take the above formula to define a function $S = f(x, y)$.

(a) (6 points) Find formulas for f_x, f_y and show that $x = 0$, $y = 1$ is a critical point.

(b) (6 points) Using the formulas below, classify the critical point in part (a) with the second derivative test. Show all work in your calculations.

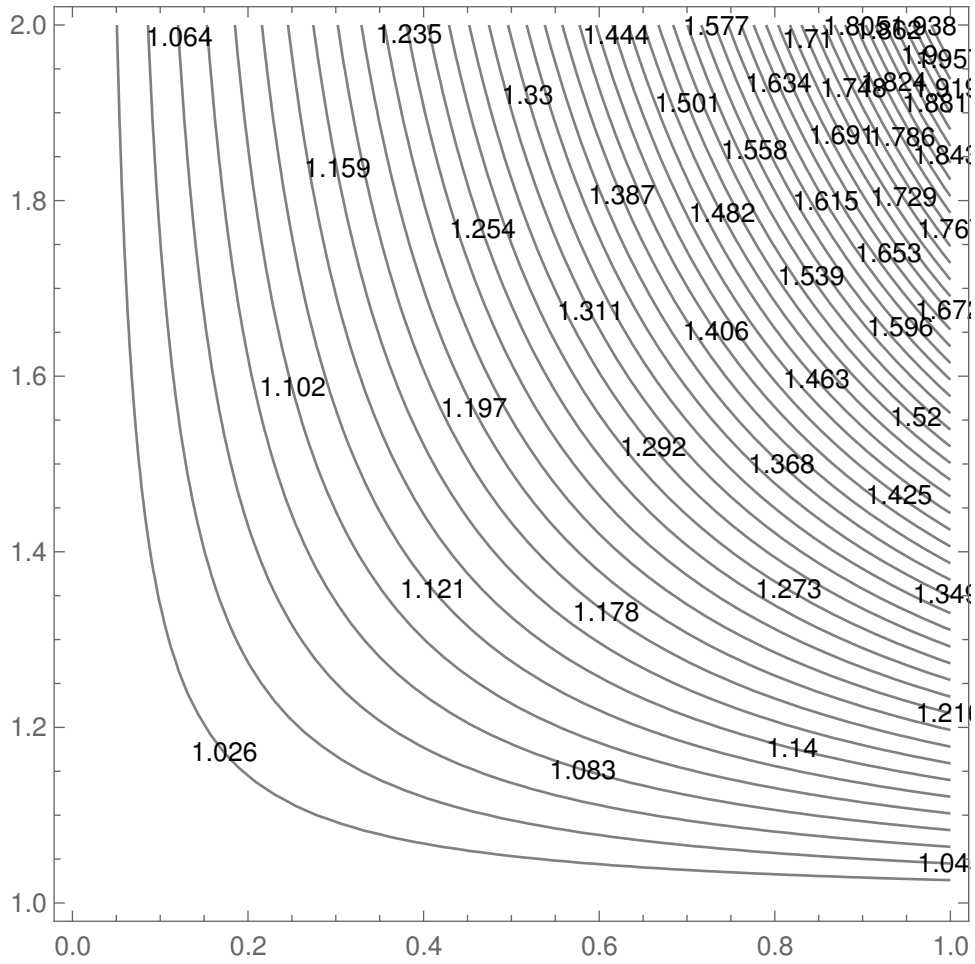
$$f_{xx} = \frac{2\left(\frac{1}{y} - 1\right)^2}{\left(\frac{x}{y} - x + 1\right)^3}$$

$$f_{xy} = \frac{1}{y^2\left(\frac{x}{y} - x + 1\right)^2} - \frac{2x\left(\frac{1}{y} - 1\right)}{y^2\left(\frac{x}{y} - x + 1\right)^3}$$

$$f_{yy} = \frac{2x^2}{y^4\left(\frac{x}{y} - x + 1\right)^3} - \frac{2x}{y^3\left(\frac{x}{y} - x + 1\right)^2}$$

(part (c) on next page)

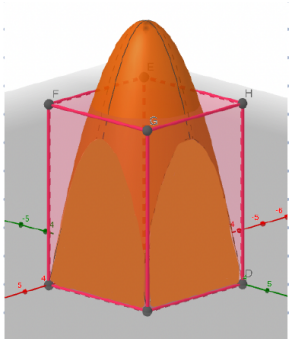
In context, it only makes sense to consider values of (x, y) where x is a proportion $(0 < x < 1)$ and y represents an actual speedup $(y > 1)$. The diagram below shows several contours for $f(x, y)$ in this domain.



(c) (6 points) Compute a Riemann sum approximation for $\iint_R f(x, y)\,\mathrm{d}A$, where R is the rectangle $0 \le x \le 1$, $1 \le y \le 2$, using four equal sub-rectangles. Do not attempt to evaluate the double integral exactly.

(The true value is $\ln(4) - 1 + \frac{\pi^2}{12} \approx 1.20876$, but you will only be graded on your methodology and presentation, not how close you come to this value.)

Problem 6. (18 points) You receive a candle in a box for your birthday, as pictured below:



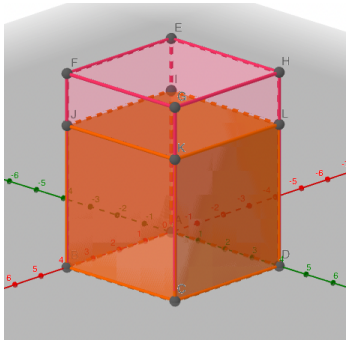
The box has a 4" $\times$ 4" square base, and the surface of the candle is described by the equation

$$f(x, y) = -(x-2)^2 - (y-2)^2 + 8$$

where $0 \leq x \leq 4$ and $0 \leq y \leq 4$.

(a) (12 points) Find the volume of the candle. To receive full credit, be sure to show all work in your calculations.

(b) (6 points) You leave the candle in your car and when you return the wax has melted into the following prism:



After the wax has melted, what is the height of the wax in the box?