
Problem 1.

(a) (12 points) Find an equation of the form $ax+by+cz=d$ for the plane passing through the point $(-2,1,4)$ that is perpendicular to the line with parametric equations $x=2t$, $y=3t-1$, $z=5-t$.

(b) (12 points) What is the distance of the point $P(1,1,1)$ from the plane in part (a)?

Problem 2. Let $F(x,y,z)=e^{2x+y-z}+xyz$.

(a) (12 points) Find the tangent plane to the surface $F(x,y,z)=1$ at the point $(0,1,1)$ and use this to estimate a value of x_0 for which the point $(x_0,1.1,0.95)$ lies on the surface.

(b) (12 points) Suppose the curve $\mathbf{r}(t)$ satisfies $\mathbf{r}(0)=\langle 1,-1,1\rangle$ with $\mathbf{r}'(0)=\langle -1,1,2\rangle$. Find the value of $\frac{\mathrm{d}}{\mathrm{d}t}[F(\mathbf{r}(t))]$ when $t=0$.

Problem 3. Consider the function $f(x,y)=xy^2+\frac{x^3}{12}-x$ on the domain

$$D=\big\{(x,y)\in\mathbb{R}^2 \mid x\in[-3,3],\ y\in[-2,2]\big\}$$

(a) (12 points) Find the critical points of $f(x,y)$ contained in the interior of D and classify each as a local maximum, local minimum, or saddle point.

(b) (12 points) Find the absolute maximum and minimum value of $f(x,y)$ on the domain D.

Problem 4.

(a) (12 points) Switch the order of integration and evaluate:

$$\int_0^{\pi/2}\int_x^{\pi/2}\frac{\sin y}{y}\,\mathrm{d}y\,\mathrm{d}x.$$

(b) (12 points) Rewrite

$$\int_1^2\int_{x^2}^5\int_0^{5-y} f(x,y,z)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

as a triple integral having order of integration $\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$.

Problem 5. Let S be the solid bounded above by the sphere $x^2 + y^2 + z^2 = 16$ and below by the paraboloid $z = \frac{1}{6}(x^2 + y^2)$.

(a) (12 points) Set up a triple integral in spherical coordinates for the volume of S.
(You do not need to evaluate the integral.)

(b) (12 points) Repeat part (a) using cylindrical coordinates.
(You do not need to evaluate the integral.)

Problem 6. (20 points) Consider the intersection of the elliptic paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$. Let C be the curve traveling along this intersection from the point $(0, 0, 0)$ to the point $(1, 1, 5)$. Evaluate the line integral

$$\int_C (3x^2 + yz)\,\mathrm{d}x + (3y^2 + xz)\,\mathrm{d}y + (3z^2 + xy)\,\mathrm{d}z$$

(Hint: You may use the Fundamental Theorem for Line Integrals, or compute the integral directly.)

Problem 7. (20 points) Let M be the parametric surface given by $x = uv$, $y = uv$, $z = u^2 - v^2$, for $0 \le u \le 1$ and $-1 \le v \le 1$. Find

$$\iint_M x^2 + y^2\,\mathrm{d}S.$$

Problem 8. (20 points) Evaluate the line integral

$$\oint_C (x^2y + xy + e^x)\,\mathrm{d}x + (\tfrac{1}{3}x^3 + xy)\,\mathrm{d}y$$

where C is the closed, piecewise-linear path from $(4, 4)$ to $(1, 1)$ to $(4, 1)$ and back to $(4, 4)$.

(Hint: You may find it convenient to use one of the theorems discussed in this course.)

Problem 9. (20 points) Consider the vector field

$$\mathbf{F} = (-2yz)\,\mathbf{i} + y\,\mathbf{j} + (3x)\,\mathbf{k}.$$

Calculate

$$\iint_S \operatorname{curl} \mathbf{F} \cdot \mathrm{d}\mathbf{S},$$

where S is the part of the paraboloid $z = 5 - x^2 - y^2$ that lies above the plane $z = 1$, with unit normal vector pointing upward (positive z-component).

(Hint: You may find it convenient to use one of the theorems discussed in this course.)

INSTRUCTIONS

- Write the **problem number** and your **USC ID** on every page.
- Responses should include step-by-step work with relevant justification, taking note of the problem instructions. Answers without much explanation may not receive much credit. State the names of any theorems you use.
- Please write clearly and legibly, and when asked to perform a calculation, put a box around your $\boxed{\text{final answer}}$.
- You are allowed to consult one page of notes (front and back) that you have prepared. No calculators or other mathematical software is allowed. No use of the internet is allowed aside from Zoom.
- At the end of the examination, you will have **20 minutes** to scan your solutions to a pdf file and upload to Gradescope. The course name is **Math 226 Final Exam Spring 2020** (entry code **M4WG6V**)
- Once your scan is uploaded, complete the **page-matching step** before submitting your work. Ensure that your scan is legible.
- If any connection issue occurs, email your instructor immediately, and be ready to record a video of yourself completing rest of the exam with your laptop or phone to send to your instructor.