Math 226 Final Exam
December 11th, 2019

Directions: Before starting the exam:

- Write down your name and your student ID.
- Check the box next to the class for which you are registered.
- Read the following rules and then sign.

You must **show all of your work and justify your methods** to obtain full credit. Simplify your final answers and then circle them. **If you use scratch paper, please make sure that all of the work that you want graded is included in the relevant portion of the exam packet.** You have been provided with plenty of blank space in the exam packet for this reason (you have two pages for each of the last two problems).

No calculators are allowed, but you may use the double sided HANDWRITTEN sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Name (please print): ______________________

Signature: ______________________

Student ID: ______________________

☐ N. Bottman 9 AM	☐ N. Bottman 10 AM	☐ R. Sacker 11 AM
☐ N. Tiruviluamala 10 AM	☐ N. Tiruviluamala 11 PM	☐ A. Mazel-Gee 12 PM
☐ W. Ozanski 12 PM	☐ W. Ozanski 1 PM	

1 (10 pts)	6 (15 pts)
2 (10 pts)	7 (10 pts)
3 (10 pts)	8 (12 pts)
4 (10 pts)	9 (13 pts)
5 (10 pts)	

100 Points Total

1. (10 points) Consider the two parallel planes $x + y + 3z = 7$ and $x + y + 3z = 9$.

   (a) (5 points) Find the distance between these two planes.

(b) (5 points) Consider the line parameterized by $\mathbf{r}(t) = \langle 1, -t, t \rangle$, $t \in \mathbb{R}$. This line intersects the two planes defined on the previous page at two respective points P and Q. Find the length of the line segment $\overline{PQ}$.

2. (10 points) The temperature near Planet X is given by the function

$$T(x, y, z) = x\cos(y)e^{z-1}.$$

(a) (5 points) At the point $P = \big(-1, 0, 1\big)$, in which direction is the temperature most rapidly **decreasing**? For your answer, provide a vector of unit length.

(b) (5 points) A spaceship moves through space, with position at time t given by

$$x = -te^{t-1}, \qquad y = \sin(\pi t), \qquad z = t.$$

At $t = 1$, what is the rate of change of the temperature experienced by the spaceship? (The temperature T is defined on the previous page.)

3. (10 points) Consider the surface S in $\mathbb{R}^3$ defined by

$$xyz + x^3 + y^3 + z^3 = 6.$$

Let P be the point on S with coordinates $P = (-1, 1, 2)$.

(a) (5 points) Find an equation of the tangent plane to S at P.

(b) (5 points) Find parametric equations of the line of intersection of the tangent plane from (a) with the plane $y = 1$.

4. (10 points) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x, y) = 6x^2 + 16y^3 + 48xy - 24x + 48y + 7.$$

(a) (5 points) Find the critical points of f.

(b) (5 points) Classify the above critical points as local maxima, local minima, or saddle points.

5. (10 points) A box is to be constructed with a volume of 256 cubic inches. The box has four sides and a bottom, but no top. What are the dimensions of a box like this that has the smallest surface area?

6. (15 points) Let E be the solid region in $\mathbb{R}^3$ located in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) inside the cylinder $x^2 + y^2 = 4$ and below the paraboloid $z = x^2 + y^2 + 1$.

   (a) (6 points) Describe E using cylindrical coordinates $\big(r, \theta, z\big)$. In other words, fill in the blanks below:

$$
\begin{gathered}
\_\_ \leq r \leq \_\_ \\
\_\_ \leq \theta \leq \_\_ \\
\_\_ \leq z \leq \_\_
\end{gathered}
$$

(b) (9 points) Let S be the boundary surface of E, oriented outwards. (Here, E is the solid region defined on the previous page.) Evaluate

$$\iint_S \boldsymbol{F} \cdot d\boldsymbol{S},$$

where $\boldsymbol{F} = \langle xy^2 - 3xz^2, yx^2, z^3 \rangle$.

7. (10 points) Consider the vector field $\boldsymbol{F}(x, y, z) = \langle y^2, 2xy, z^2 \rangle$.

   (a) (4 points) Determine whether or not $\boldsymbol{F}$ is conservative. If it is, find a function f such that $\boldsymbol{F} = \nabla f$.

(b) (6 points) For the vector field $\boldsymbol{F}$ defined on the previous page, evaluate $\int_C \boldsymbol{F} \cdot d\mathbf{r}$, where C is the curve given by $\mathbf{r}(t) = \langle t^3 - 1, 2 - t^2, t \rangle$, for $t \in [0, 1]$.

8. (12 points) Calculate

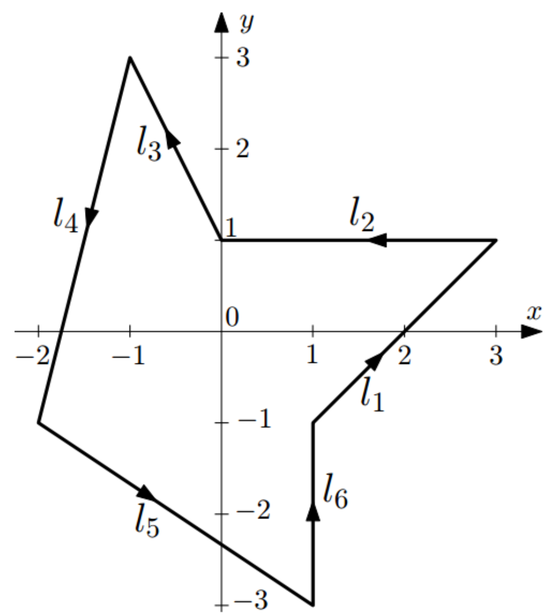
$$\int_{l_2} \boldsymbol{F} \cdot d\mathbf{r} + \int_{l_3} \boldsymbol{F} \cdot d\mathbf{r} + \int_{l_4} \boldsymbol{F} \cdot d\mathbf{r} + \int_{l_5} \boldsymbol{F} \cdot d\mathbf{r} + \int_{l_6} \boldsymbol{F} \cdot d\mathbf{r},$$

where

$$\boldsymbol{F} = \langle -y, x \rangle,$$

and the oriented line segments $l_1, \cdots, l_6$ are as in the figure below. Note that $\int_{l_1} \boldsymbol{F} \cdot d\mathbf{r}$ is not included in the above sum.

Hint: You may use without justification the fact that the area enclosed by the figure is 12.



9. (13 points) Evaluate $\displaystyle\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$, where $\boldsymbol{F} = \langle 6x + e^{\sin^2 x}, \arctan(2y - y^2), 6z - x^2\rangle$ and C is the triangle with vertices $(0,0,3)$, $(0,2,0)$, and $(4,0,0)$ oriented as in the figure below.