
Problem 1. Find the following antiderivatives:

(a) $F(x) = \int x^2 e^{-x}\, dx$

(b) $G(x) = \int \tan^3(x)\, dx$

(c) $H(x) = \int \frac{x^2}{(9+x^2)^{\frac{3}{2}}}\, dx$

Problem 2. Use any method to determine if each of the following integrals converges or diverges.

(a) $I = \int_0^{+\infty} \frac{e^{\sin(x)}}{x^2+x+1}\, dx$

(b) $J = \int_0^{1/2} \frac{1}{x-x^2}\, dx$

Problem 3.

(a) Determine the partial fraction decomposition of the rational function

$$f(x) = \frac{-2x^2+1}{(x+1)(x^2+x+1)}.$$

(b) Calculate the antiderivative $F(x)$ of the rational function

$$f(x) = \frac{-2}{3x+1} + \frac{1}{(5-7x)^3} - \frac{x+3}{x^2-2x+5}.$$

Problem 4. Determine if each series is absolutely convergent (ACV), conditionally convergent (CCV) or divergent (DV). You may use any method but you must **clearly identify which tests you use**.

(a) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{\sqrt[3]{n^2+n}}$

(b) $\sum_{n=1}^{+\infty} \frac{(-1)^n(n+1)(3^n)}{2^{2n+1}}$

(c) $\sum_{n=1}^{+\infty} \frac{(n+1)^n}{n!}$

Problem 5. Let $f(x) = e^{x-1}\ln(x)$.

(a) Use the Taylor polynomial of degree 2 centered at $x = 1$ to estimate $f(1.01)$.

(b) Use Part (a) to find an estimate for the value of $\int_1^{.98} e^{x-1}\ln(x)\,dx$.

Problem 6. Find the **length** of the curve $x = \frac{y^4}{8} + \frac{1}{4y^2}$ from $y = 1$ to $y = 2$.

Problem 7. Consider the region $\mathcal{R}$ contained in the 4th quadrant, bounded by the curves $x = 0$, $y = 0$, $y = \ln(x)$. Calculate the **volume** of the solid E obtained by rotating the region $\mathcal{R}$ around the line $x = -1$. A sketch may come in handy. You must fully justify your findings.

Problem 8. Consider the function $f(x)$ defined by

$$f(x) = \sum_{n=0}^{+\infty} (-1)^{n+1} \frac{(2x-6)^n}{4^n}$$

(a) Determine the interval of convergence of $f(x)$.

(b) Determine the value of the 5th derivative of $f(x)$ at $x = 3$.

(c) Express $f(x)$ in terms of x, without using a summation. You must fully simplify your answer for full credit.

Problem 9. Consider the sequence (a_n) defined recursively by

$$\begin{cases} a_0 = -1 \\ a_{n+1} = \dfrac{-2}{2a_n + 5}, & \text{for } n \geq 0 \end{cases}.$$

You may freely use the following **fact**: (a_n) is always contained in $\left(-\frac{5}{2}, \frac{7}{2}\right)$.

(a) Compute the values of a_0, a_1, a_2.

(b) Use mathematical induction to determine whether (a_n) is increasing or decreasing.

(c) Show that (a_n) is convergent and determine the value of its limit. You must justify your findings.

(d) Determine the convergence of the series $\sum_{n=0}^{+\infty} a_n$. Justify your answer.

INSTRUCTIONS

i) You should submit each problem on a different page; i.e., no two problems should appear on the same page.

ii) You must include both the **problem number** and your **USC ID** on **every page**.

iii) You must show all your work and carefully state and justify your methods to obtain full credit. Answers must be neat, organized and unambiguous.

iv) **No calculator of any kind; no use of the internet is allowed aside from Zoom**. USC considers cheating as a very serious offense; the minimum penalty is failure for the course. Cheating does include failing to stop writing when told at the end of the examination.

v) You will be given **15 minutes** to neatly scan and submit your work at the end of the examination.

   You must submit your work through Gradescope; the course name is **"Math 126 FINAL EXAM"**. Once your scan is uploaded, you must complete the **page-matching step** before submitting your work. You are responsible for submitting a legible scan of high quality. Failure to properly submit your work according to the above guideline will result in penalties.

vi) If any connection issue with Zoom or Gradescope occurs, you must email your instructor **immediately** and be ready to record your activity with your phone or alternative device. You should stay in contact with your instructor through the end of the exam. You will be asked to send the video file to your instructor.