

Spring 2018 Math 126 Final

Please Print:

Your instructor's name:

Last name:

First Name:

Signature:

ID#:

Problem	Score
1	/ 18
2	/ 24
3	/ 10
4	/ 16
5	/ 16
6	/ 16
7	/ 16
8	/ 24
9	/ 16
10	/ 24
Total	/ 180

INSTRUCTIONS:

1. NO Calculators allowed. Cheating is not tolerated.
2. Clearly indicate your final answers by circling them.
3. Show all your work. Unsupported answers will not receive credit.
4. In general you do not need to “simplify” your answers, but you will need to evaluate simple numbers.
5. Point values are labeled and there are 180 total points possible.

Good luck!

1. [18 points]

Evaluate the following limits

(a) [6 points] $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - x - 1}{x^2}$.

(b) [6 points] $\lim_{x \rightarrow \infty} (xe^{1/x} - x)$.

(c) [6 points] $\lim_{x \rightarrow 0^+} x^{\sin x}$.

2. [24 points]

Evaluate each anti-derivative:

(a) [8 points] $\int \sqrt{1 - 4x^2} \, dx$

(b) [8 points] $\int \sin^2 x \cos^3 x \, dx$

(c) [8 points] $\int x^3 \sqrt{1+x^2} dx$

3. [10 points]

- (a) [**6 points**] Find the partial fractions decomposition of $\frac{x+5}{x^2-5x}$.
(Solve for the coefficients.)

- (b) [**4 points**] Evaluate $\int \frac{x+5}{x^2-5x} dx$.

4. [16 points] In each part below, decide whether the improper integral converges or diverges. Circle the correct answer and provide justification.

(a) [8 points] $\int_2^{\infty} \frac{1}{\sqrt{x^2 - 1}} dx$

CONVERGES

DIVERGES

Justification:

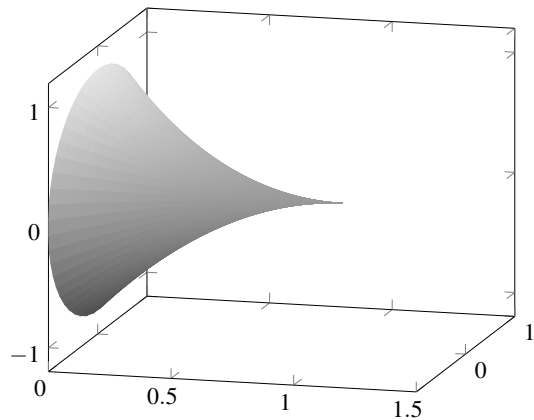
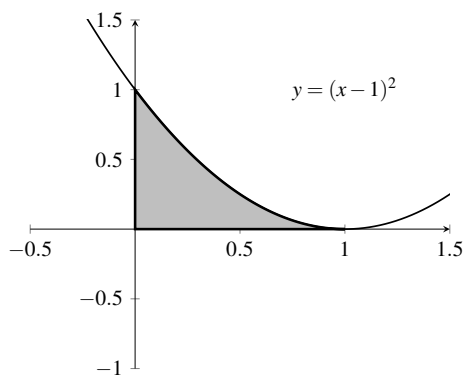
(b) [8 points] $\int_0^1 \frac{\cos(t)}{\sqrt{t}} dt$

CONVERGES

DIVERGES

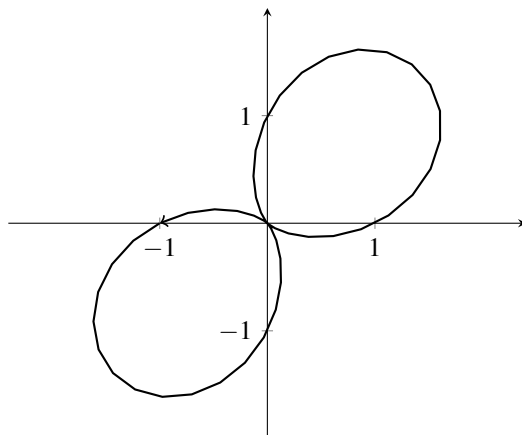
Justification:

5. [16 points] The region bounded by the curve $y = (x - 1)^2$, the x -axis, and the y -axis is rotated around the x -axis. What is the volume of the resulting solid?



6. [16 points] A tank has the shape of an inverted circular cone with height 12 m and base radius 5 m. It is filled with oil to a height of 8m. Find the work required to empty the tank by pumping all of the oil to the top of the tank.
(The density of oil is 900 kg/m^3 . Use g for the gravitational constant.)

7. [16 points] Consider the curve (pictured below) given in polar coordinates by $r = 1 + \sin(2\theta)$.



Find the area of one of the loops pictured above.

8. [24 points]

For each of the following series, determine if it is absolutely convergent, conditionally convergent, or divergent. Clearly state the method(s) you use.

(a) [8 points]
$$\sum_{n=0}^{\infty} \left(\frac{2^{3n}}{\pi^{2n}} + \frac{n}{\sqrt[3]{n^7 + 1}} \right).$$

(b) [8 points]
$$\sum_{n=0}^{\infty} \frac{(n!)^3}{(3n)!}.$$

(c) [8 points] $\sum_{k=2}^{\infty} \frac{(-1)^k}{k\sqrt{\ln k}}$.

9. [16 points] Consider the following power series:

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(n+1)(x+2)^n}{3^{n+1}}$$

Find the radius of convergence and interval of convergence of the series.

10. [24 points] Let $f(x) = x^4 e^{-x^2}$.

(a) [**8 points**] Find the Maclaurin series of $f(x)$ and its radius of convergence.

(b) [**8 points**] Find $f^{(2018)}(0)$.

- (c) [**8 points**] Evaluate $\int_0^1 f(x) dx$ to within 0.001, justifying the precision.
(You can leave your answer as a finite sum.)