Problem 1. Evaluate each the following limits, including $\pm\infty$, or show the limit does not exist.

(a) $\lim\limits_{x\to 0} \dfrac{x - \sin(x)}{x^3}$.

(b) $\lim\limits_{x\to 1^-} \cos^{-1}(x)\tan\left(\dfrac{\pi x}{2}\right)$.

Problem 2. Evaluate the following integrals:

(a) $\displaystyle\int x^5 \ln x\, dx$.

(b) $\displaystyle\int \frac{x^2}{\sqrt{9 - x^2}}\, dx$.

Problem 3. Consider the following improper integrals.

(a) Determine whether $\displaystyle\int_2^9 \frac{z}{\sqrt{z-2}}\, dz$ converges or diverges by evaluating it.

(b) Use the comparison test on $\displaystyle\int_{-\infty}^0 e^{-x^2}\, dx$ to demonstrate whether it converges or diverges.

Problem 4. Consider the region bounded by the curves $y = \cos^2(x)$, $y = 0$, $x = 2\pi$ and $x = 3\pi$. *Set up, but do not evaluate,* an integral that gives the volume of the solid obtained by rotating the region about the line $x = \pi$.

Problem 5. A cylindrical tank rests on its side so that its circular base, which has a radius of 2 meters and a length of 5 meters, is perpendicular to the ground.

[PIC]

(a) Express the hydrostatic force **on one side** of the tank as an integral if the tank is filled halfway with a fluid of mass density δ kilograms per cubic meter and the acceleration due to gravity is g meters per second squared. *Do not evaluate the integral.*

(b) A crane lifts this tank to a height of 15 meters at a rate of 0.5 meters per second. As the tank is lifted, the fluid leaks out at a rate of 0.2 cubic meters per second. Express the work performed on the tank as an integral. *Do not evaluate the integral.*

Problem 6. Determine if the following series converge absolutely, converge conditionally , or diverge. Prove your answer. Clearly state your conclusion and any test(s) that you use.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5 + n^2 + 1}}$.

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n}{n \ln n}$.

Problem 7. Consider the power series: $\sum_{n=0}^{\infty} \frac{(4-9x)^n}{n+1}$.

(a) Determine a, the point where the series is centered.

(b) Show that the radius of convergence is $9/4$. That is, $R = 9/4$.

(c) Find the interval of convergence.

Problem 8. Consider the function $f(x) = x^8 \ln(1 + x^2)$.

(a) Find a power series for $f(x)$ and state its radius of convergence. *Explicitly write out the the general term, as well as first 4 terms.*

(b) Find $f^{(16)}(0)$ and $f^{(17)}(0)$.

(c) Let $T_{16}(x)$ denote the degree 16 Taylor polynomial of $f(x)$. If we use $T_{16}(1.1)$ to estimate $f(1.1)$, find an upper bound for the error $|R_{16}(1.1)|$.

(d) Find a power series for $\int f(x)\, dx$ and find its radius of convergence.

Problem 9. Consider the polar curve $r = 2 + \cos\theta$.

(a) Find all values of θ where the tangent line is horizontal for $0 \leq \theta \leq 2\pi$.

(b) Find the area enclosed by the curve.