Math 126 Final Exam
December 6th, 2017

Directions. Fill out your name, signature and student ID number on the lines below **right now**, before starting the exam! Also, check the box next to the class for which you are registered.

You must **show all your work and justify your methods** to obtain full credit. Write your final answers in the boxes provided. Simplify your answers. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$, and $\sqrt{3}$. Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed, but you may use the sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. You may have anything written on it (on both sides), but it must be written *in your own handwriting*. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes "straying eyes" and failing to stop writing when told to do so at the end of the exam.

Name (please print): ______________________

Signature: ______________________

Student ID: ______________________

☐ N. Tiruviluamala 9am	☐ N. Tiruviluamala 10am	☐ G. Welper 10am
☐ M. Hogancamp 11am	☐ N. Emerson 12pm	☐ N. Haydn 1pm

Do not write on this page below this line!

1 (15 pts)	6 (15 pts)
2 (15 pts)	7 (15 pts)
3 (15 pts)	8 (15 pts)
4 (15 pts)	9 (15 pts)
5 (15 pts)	10 (15 pts)

150 points total

Problem 1 (15 points) Evaluate the following limits.

(a) $\lim_{x \to 0} \frac{\tan^{-1}(x)}{\sin x}$

(b) $\lim_{x \to \infty} \left(\sqrt{2x^2 + x + 5} - \sqrt{2x^2 + 5} \right)$

(c) $\lim_{x \to \infty} \left(\cos\left(\frac{1}{x}\right) \right)^x$

Problem 2 (15 points) Compute the following antiderivatives.

(a) $\displaystyle\int \left(\cos^3 x\right) (\sin x)\, dx$

(b) $\displaystyle\int x^3\sqrt{4+x^2}dx$

(c) $\displaystyle\int \frac{x^2+4x+4}{(x+1)(x^2+2x+2)}dx$

Problem 3 (15 points) Determine whether each of the following improper integrals is convergent or divergent.

(a) $\displaystyle\int_0^1 \ln x \, dx$

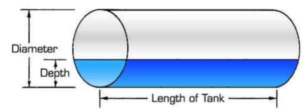
(b) $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x^6+1}}$

Problem 4 (15 points) Let T be the region bounded between the graphs of $y = x^2 + 1$ and $y = x + 3$. Set up, but **do not evaluate**, an integral representing the volume of the solids obtained by rotating T about:

(a) the line $x = 4$

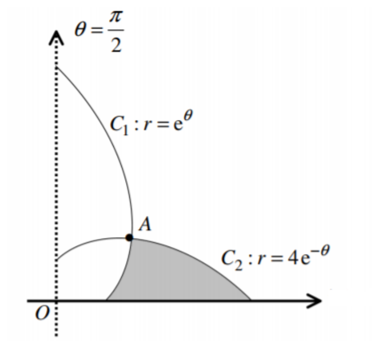
(b) the line $y = 10$

Problem 5 (15 points) A cylindrical tank with a diameter of 10 meters and length of 30 meters contains a liquid of density $\rho = 2000$ kilograms per cubic meter. The depth of the liquid is 3 meters.



Set up, but **do not evaluate**, an integral representing the the work required to pump enough liquid out to the top of the tank to reduce the depth of the remaining liquid to 1 meter.

Problem 6 (15 points) Find the area of the shaded region below. The equations of the curves C_1 and C_2 are given in polar coordinates.



Problem 7 (15 points) Determine if the following series are divergent, convergent or absolutely convergent. Justify your answers.

(a) $\sum_{n=2}^{\infty}(-1)^n \frac{n}{\ln n}$

(b) $\sum_{n=2}^{\infty} \frac{\sin(\ln(n))}{n^2+3}$

(c) $\sum_{n=1}^{\infty} \frac{e^n}{\sqrt{n!}}$

Problem 8 (15 points) Consider the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{9^n \ln(n+1)}.$$

(a) Find $f^{(16)}(1)$.

(b) Find the radius of convergence of the power series.

(c) Find the interval of convergence of the power series. Remember to check the endpoints of the interval.

Problem 9 (15 points) Consider the function

$$f(x) = x^{-\frac{1}{2}}.$$

(a) Find the second degree Taylor Polynomial $T_2(x)$ for $f(x)$ centered at $a = 4$.

(b) Use $T_2(x)$ to give an approximation for $\frac{1}{\sqrt{5}}$.

(c) Provide an upper bound for the error in your approximation from part (b).

Problem 10 (15 points) Let $f(x) = x^4 e^{x^3}$.

(a) Find the Maclaurin series of $f(x)$. Explicitly write out the first 4 terms as well.

(b) Find a series for $\int_{-1}^{0} x^4 e^{x^3}\, dx$.

(c) Find the minimal number of terms of the above series that we can use so that the error is no more than $1/100$.