Math 125 Final Exam, Spring 2020, Draft 5

Problem 1. Evaluate the following limits. Evaluate the following limits. If you are using a known limit, squeeze theorem, or any other technique, show all of your work and any supporting steps to receive credit. Any answer with limited or no supporting work, or that use L'Hopital's rule, will receive no credit.

a. $\lim\limits_{x \to \infty} \dfrac{\sin(\frac{2}{x}) + \frac{2}{x}}{\sin(\frac{1}{x})}$

b. $\lim\limits_{x \to 0^-} \dfrac{2^{x+1}}{\ln(x+1)}$

Problem 2. Evaluate the following integrals. Show all work and supporting steps. Answers with no work or supporting steps will receive no credit.

a. $\displaystyle\int \frac{\ln(\tan(x))}{\sin(x)\cdot\cos(x)}\,dx$

b. $\displaystyle\int_{-1/k}^{3/k} x\sqrt{kx+1}\,dx$, where k is a nonzero constant.

Problem 3. (Continuity/Dfferentiability/Product rule) Consider the following function: (We can change the function to $x^2 sin(ln(x^2)$ if we have an issue with the absolute value, and the parts below still work).

$$f(x) = \begin{cases} x^2 \sin(\ln(x^2)) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

a. Show that $f(x)$ is continuous everywhere.

b. Find $f'(x)$ for $x \neq 0$.

c. Show that $f(x)$ is differentiable at $x = 0$.

Problem 4. It's a hot day in L.A. and Carina has an ice cream cone. The ice cream is leaking into the cone at a rate of $3/2$ cm^3 per second. Given that the cone is 10 cm high, with a radius at the largest end of 3 cm, at the moment when the leaked ice cream fills half-way down the cone, what is the rate of change of the height of the liquid ice cream in the cone? (Hint: The formula for the volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$ where r is the radius of the cone, and h is the height.)

Problem 5. (Graphing) The graph of $f(x) = \dfrac{1}{1+e^{-x}}$ is given below.

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a. Prove that $f(x)$ has no vertical asymptotes.

b. Prove that $f(x)$ has horizontal asymptotes $y = 0$ and $y = 1$.

c. Prove that $f(x)$ is strictly increasing everywhere.

d. Prove that $f(x)$ has an inflection point at $x = 0$, and is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$.

Problem 6. (Riemann sum/integral). Consider the integral $\displaystyle\int_0^1 \sqrt{1+x^2}\, dx$.

a. Express the integral as a left Riemann sum with 4 sub-intervals of equal width. You may leave your answer as an unsimplified sum.

b. Is the above Riemann sum in a. greater than or less than the value of the integral? Briefly explain.

Problem 7. (MVT) Show that $\sqrt{1+x} \leq \sqrt{2} + \dfrac{x-1}{2\sqrt{2}}$ for $x \geq 1$.

Problem 8. Define $F(x) = \displaystyle\int_1^{x^3} \frac{1}{3+t^4}\, dt.$

a. Compute $F'(x)$.

b. Show that F is one-to-one.

c. Compute $(F^{-1})'(0)$.

d. Use a linear approximation to estimate $F^{-1}(0.1)$.